

# Water slope and discharge in the Amazon River estimated using the shuttle radar topography mission digital elevation model

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[1] We find that the standard deviation, hence error, of the water surface elevation data from the Shuttle Radar Topography Mission (SRTM) is 5.51 m for basin-wide, regional and local Amazon mainstem reaches. This error implies a minimum reach length of 733km in order to calculate a reliable water-surface slope. Resulting slopes are  $1.92 \pm 0.19$  cm/km for Manacapuru,  $2.86 \pm 0.24$  cm/km for Itapeua and  $3.20 \pm 0.34$  cm/km for Tupe. Manning's equation is applied with these slopes and with channel width measurements from the Global Rain Forest Mapping project synthetic aperture radar mosaics (GRFM SAR), channel depths averaged from nautical charts, and reasonable estimates of Manning's  $n$ . Resulting discharge values are 84,800 m<sup>3</sup>/s at Manacapuru, 79,800 m<sup>3</sup>/s at Itapeua, and 62,900 m<sup>3</sup>/s at Tupe averaged over the SRTM mission period. These values are within 6.2% at Manacapuru, 7.6% at Itapeua, and 0.3% at Tupe of the in-situ gage-based estimates for the same or similar time period. **Citation:** LeFavour, G., and D. Alsdorf (2005), Water slope and discharge in the Amazon River estimated using the shuttle radar topography mission digital elevation model, *Geophys. Res. Lett.*, 32, L17404, doi:10.1029/2005GL023836.

## 1. Introduction

[2] Spatial and temporal variations in discharge ( $Q$ ) and surface water storage ( $S$ ) are poorly known globally but are critical for constraining the terrestrial branch of the water cycle. For example, the stream gage density in the Amazon Basin is two orders of magnitude lower than that of the eastern U.S. even though the average annual discharge is two orders of magnitude greater in the Amazon. This monitoring effort only provides information on discharge at a few locations along the Amazon mainstem and even fewer locations on its major tributaries. Changes in  $Q$  and  $S$  play a critical role in the water cycle, particularly regarding any potential accelerations that may lead to climate changes [e.g., *Stocker and Raible*, 2005]. Furthermore,  $Q$  and  $S$  are necessary to constrain global climate models (GCMs). GCMs output precipitation, which is routed to predict discharge. For basins with channel discharge measurements, GCMs can be in error by 50% (e.g., the Mississippi Basin [*Roads et al.*, 2003]) and in some cases by as much as 100% (e.g., the Nile River basin [*Coe*, 2000]).

[3] Remote sensing has the potential to offer the spatial and temporal coverage necessary to measure water fluxes globally [e.g., *Alsdorf et al.*, 2003; *Alsdorf and Lettenmaier*, 2003]. Most water flux estimation methods use regression-based relationships between remotely measured inundated area and in-situ measured discharge to predict fluxes [e.g., *Smith*, 1997]. However, this approach does not work well in environments where small changes in water heights yield little change in surface area yet significant changes in flow. For example, an elevation change of 5 cm over 24 hours in just 1/25 of the Amazon's flooded 750,000 km<sup>2</sup> can be equivalent to the average discharge of the Mississippi River, yet the change in flooded area is practically undetectable at any spatial resolution. Instead, *Alsdorf and Lettenmaier* [2003] have called for methods that measure surface water hydraulics, particularly elevations, to estimate  $Q$  and  $S$ . In this paper, we provide the first documented use of the SRTM DEM to estimate stream channel water elevations, slopes, and discharges (C-band, 3 arc second data are used).

## 2. Flow Hydraulics

[4] Manning's equation (equation (1)) provides an empirical estimate of velocity,  $V$  in m/s, for open-channel, uniform flow conditions [*Albertson and Simons*, 1964],

$$V = (k/n) \left( R^{2/3} S^{1/2} \right) \quad (1)$$

where  $k$  is a conversion factor =  $1.0 \text{ m}^{1/3} \text{ s}^{-1}$ ,  $n$  = Manning's roughness coefficient,  $R$  = hydraulic radius (m) =  $A/P$ ,  $A$  = cross-sectional area in m<sup>2</sup>,  $P$  = wetted perimeter in m, and  $S$  = slope (assumed to be the water surface gradient) in m/m. Velocity multiplied by channel cross-sectional area ( $A$ ) yields  $Q$ . To calculate discharge of the Amazon mainstem, we use equation (1) and water surface slopes derived from the C-band SRTM DEM to estimate  $V$ .

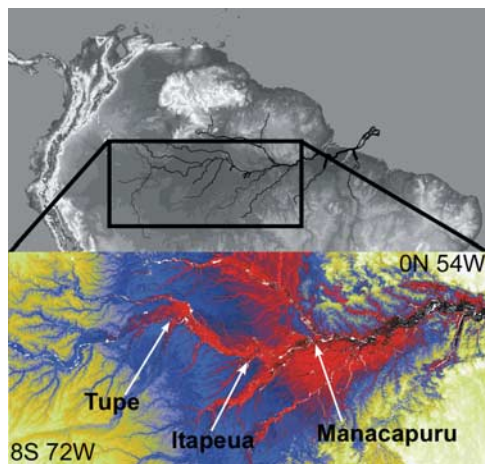
## 3. Measurement Errors: Ancillary Data

[5] Each parameter in the Manning equation has an associated measurement error. Channel width, depth, and Manning's  $n$  are needed, in addition to slope, to estimate  $V$ . Of the geometric variables used in the Manning equation, only bathymetry cannot be remotely sensed (see below). We selected three Amazon mainstem reaches centered on the small towns of Manacapuru, Itapeua and Tupe (Figure 1) because of collocated, existing in-situ gage data and field campaign data from the Carbon in the Amazon River Experiment (CAMREX) [*Richey et al.*, 1990].

[6] Channel depths at the study locations were estimated from nautical charts [*Marinha do Brasil*, 1969], which used soundings collected from boat cruises along the Amazon

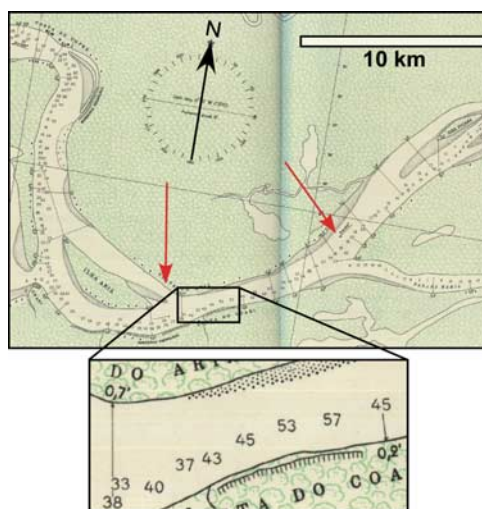
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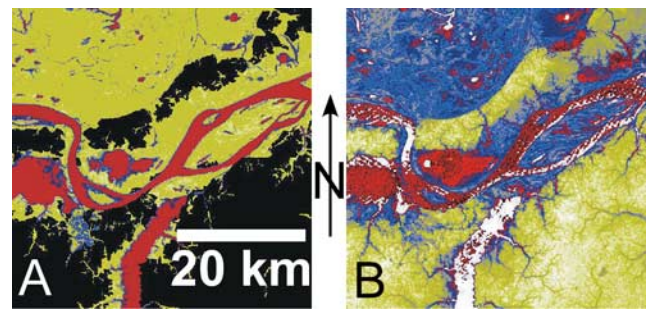


**Figure 1.** Location and extent of the SRTM DEM used in this study.

mainstem. These charts do not provide explicit channel cross-sections, but because of acquisition pathways and stream geomorphology, a reasonable value for local depths can be estimated. At each study location, an average depth was calculated from soundings extending upstream and downstream of the local gage (Figure 2). To avoid depth errors associated with multiple channels around islands (e.g., erroneous shallowing and doubling of the overall channel width), averaging was conducted over reaches where the mainstem formed a single thread. For example, the average depth at Itapeua is 34.3 m (Table 1b, standard deviation,  $\sigma$ , is 11.3 m) over a 10.1 km reach and is 22.7 m ( $\sigma = 6.3$  m) over an 8.0 km reach at Tupe. Both a shallow and a deep profile are present at Manacapuru, with the shallow profile containing many more soundings than the deep profile. To correct this imbalance, the mean depth of the shallow profile (17.5 m,  $\sigma = 6.4$  m, 17.7 km) was averaged with the mean depth of the deep profile (22.8 m,  $\sigma = 9.4$  m, 13.7 km) to yield a Manacapuru depth of 19.4 m ( $\sigma = 7.6$  m). For comparisons, the average CAMREX cross-



**Figure 2.** Close-up of Brazilian nautical charts used to estimate channel depth at the Itapeua study location. Arrows mark the reach of depths averaged.



**Figure 3.** (a) GRFM classification and (b) C-Band SRTM DEM at Itapeua. In the GRFM classification the maroon color shows areas of open water. In the DEM, colored open-water pixels represent actual water-surface elevations and white open-water pixels represent no-data values.

sectional depths near the gage locations are 22.4 m at Manacapuru (range is 16.6 m–26.0 m), 35.7 m at Itapeua (25.8 m–40.0 m), and 20.9 m at Tupe (16.6 m–24.0 m). The close agreement between the values obtained using our simple averaging method and the in-situ CAMREX measurements suggests that depth errors are much smaller than implied by the standard deviations. In February the annual Amazon flood wave is near its peak at Tupe, is between mid-rising and peak conditions at Itapeua, and is at mid-rising water heights at Manacapuru. Thus the average CAMREX depth is assumed to be the most correct at Manacapuru, a depth value between the greatest depth and average at Itapeua is assumed correct, and the largest depth is considered correct for Tupe. This results in the following depth errors:  $\pm 3.0$  m at Manacapuru (22.4 m–19.4 m),  $\pm 3.6$  m at Itapeua (37.9 m–34.3 m), and  $\pm 1.3$  m at Tupe (24.0 m–22.7 m).

[7] Channel widths at Manacapuru, Itapeua and Tupe were extracted from the classification data of Hess *et al.* [2003]. They classified the GRFM SAR mosaic [Rosenqvist *et al.*, 2000] using decision tree schemes to separate radar backscatter pixel amplitudes as indicators of nine ecological habitats. The classification accuracy is greater than 90%. This classification distinguishes open-water pixels from surrounding pixels making it possible to measure width across the channel at gage locations (Figure 3). An additional source of error is contributed by the mixed-pixel problem. Therefore, we suggest that the width measurement error is  $\pm 1$  pixel ( $\pm 92.5$  m).

**Table 1a.** Estimated Discharge and Related Errors for Manacapuru<sup>a</sup>

	Width, m	Depth, m	Slope, m/km	Manning's n	V, m/s	Q srtn, m <sup>3</sup> /s	Q Observed for ANA 2000, m <sup>3</sup> /s	Error, %
Max	3593	22.4	0.021	0.028	1.29	103794	90500	14.69
Avg	3500	19.4	0.019	0.025	1.25	84849	90500	-6.24
Min	3408	16.4	0.017	0.022	1.20	67174	90500	-25.77

<sup>a</sup>Input parameters to the Manning equation, resulting velocity (V) and discharge (Q). Remotely sensed parameters of slope and channel width, in-situ depth, and Manning's n are shown. The average of each parameter represents the measured value, while the maximum and minimum represent the highest and lowest value of that parameter based on associated measurement errors. Q is compared to observed values from in-situ gage data at Manacapuru and Itapeua, and from CAMREX data for Tupe.

**Table 1b.** Estimated Discharge and Related Errors for Itapeua<sup>a</sup>

	Width, m	Depth, m	Slope, m/km	n	V, m/s	Q srtn, m <sup>3</sup> /s	Q Observed, m <sup>3</sup> /s		% Error	
							DNAEE 9173–91	DNAEE 1981	DNAEE 1973–91	DNAEE 1981
Max	1159	37.9	0.031	0.028	2.15	94435	83106	74157	13.63	27.35
Avg	1066	34.3	0.029	0.025	2.18	79760	83106	74157	-4.03	7.55
Min	974	30.7	0.026	0.022	2.18	65201	83106	74157	-21.54	-12.08

<sup>a</sup>As in Table 1a footnotes.

[8] The width-to-depth ratio of the mainstem at all three study locations is large (Tables 1a–1c), thus errors associated with the channel cross-sectional shape (e.g., rectangular, parabolic, etc.) are small. Such errors are essentially incorporated in the channel depth.

[9] *Albertson and Simons* [1964] suggest that Manning's roughness coefficient for alluvial, sand bedded channels with no vegetation (i.e., the Amazon mainstem) ranges from 0.018 to 0.035. They also suggest that rivers in fair condition, with little moss growth, have an *n* of 0.025, in contrast to winding natural streams in poor condition, with considerable moss growth, having an *n* of 0.035. We use 0.025 as the typical *n* value with an error, albeit subjective, of  $\pm 0.003$ .

#### 4. Water Surface Height Errors

[10] The SRTM DEM yields elevations over open water surfaces (Figure 3) because wind roughening and wave action can produce backscattering at the large look angles (30° to 58°) and C-band frequency of the mission. The DEM has a relative vertical error of 5.5 m over land with 90% significance [*Rodriguez*, 2005]. Errors over open water, however, are unknown, thus we use basin-wide, regional, and local approaches to investigate this error along the mainstem Amazon.

[11] Basin-wide, mainstem elevations were extracted from the SRTM DEM and combined with flow distances (Figure 4). Polynomials of increasing order, from 1st to 10th, were fit to the entire 3000 km, basin-wide reach. The goodness of fit shows marked improvement from 1st to 3rd order, whereas subsequent orders yield little improvement, thus we select a 3rd order polynomial. The standard deviation after subtracting the polynomial from the 3000 km reach of SRTM water surface heights is 5.51 m.

[12] Polynomials were fit to data for a series of 300 km to 1000 km reaches to determine the potential regional influences on the SRTM derived water surface heights. The goodness of fit criteria was used to select the optimal polynomial order for 300 km, 500 km, 700 km and 1000 km reaches centered on the individual gages at Manacapuru, Itapeua and Tupe. Reach lengths were centered on the study locations, rather than extending only upstream, because polynomials are poorly constrained at their end points. After

removal of the various regional polynomial trends, standard deviations average 5.58 m (5.32 m to 6.04 m).

[13] The possibility of local influences on SRTM water-surface heights were investigated using linear trends along small reach lengths, starting with 4 km and incrementing by 4 km. Because the C-band DEM does not provide heights at all locations along the mainstem, the final reach lengths investigated were 100 km at Manacapuru, 80 km at Itapeua and 56 km at Tupe. The mean standard deviation of the data, after removing the linear trends, is 6.04 m at Manacapuru (range 5.99 m to 6.15 m), 5.56 m at Itapeua (5.21 m to 6.04 m), and 4.10 m at Tupe (3.91 m to 4.29 m).

[14] Given these trend analyses along the Amazon mainstem, we find little geographic variation in the standard deviation of the SRTM C-Band DEM water surface heights. The overall value of  $\sigma$  is 5.51 m found by simple averaging of the results from the various trends. Unlike the land surface, elevations along the water surface are expected to have little variation, especially for the broad, lowland Amazon River. Thus, after removing the simple trends from the SRTM heights, the resulting  $\sigma$  is a strong indicator of height error. We use  $\pm 5.51$  m as the height error. Water bodies located poleward of the equatorial Amazon measurements should have reduced height errors because of the increased averaging from a greater number of data takes collected during the SRTM.

#### 5. Water Surface Slopes

[15] To calculate a reliable slope from SRTM, reach lengths need to extend sufficient distances to accommodate the height errors (Figure 5). We developed a simple relationship (equation (2)) to determine the appropriate reach length (RL).

$$2\sigma/\text{RL} = S_{\min} \quad (2)$$

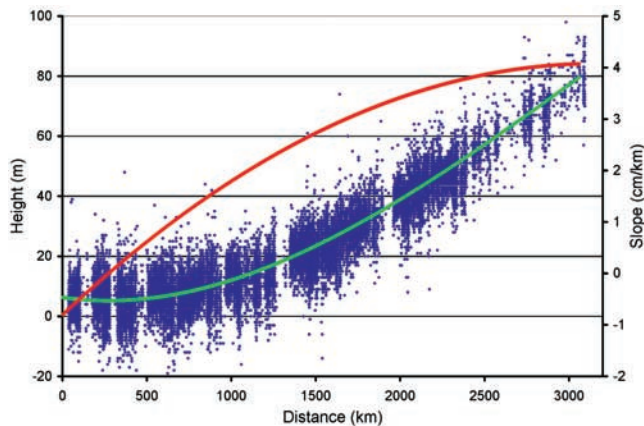
Based on Topex/POSEIDON radar altimetry over the central Amazon Basin, *Birkett et al.* [2002] found a minimum slope,  $S_{\min}$ , along the mainstem of 1.5 cm/km. Using our derived  $\pm 5.51$  m height error, we find an appropriate RL of 733 km.

[16] Slopes were calculated for the three study locations by fitting a 2nd order polynomial to the SRTM water surface heights collected from 733 km reaches centered on each

**Table 1c.** Estimated Discharge and Related Errors for Tupe<sup>a</sup>

	Width, m	Depth, m	Slope, m/km	n	V, m/s	Q srtn, m <sup>3</sup> /s	Q Observed for CAMREX, m <sup>3</sup> /s	% Error
Max	1650	24.0	0.035	0.028	1.72	68277	63100	8.20
Avg	1557	22.7	0.032	0.025	1.78	62899	63100	-0.32
Min	1465	21.4	0.029	0.022	1.85	58007	63100	-8.07

<sup>a</sup>As in Table 1a footnotes.



**Figure 4.** Amazon River basin-wide water-surface SRTM C-band heights (blue dots). A 3rd order polynomial is fit to the data (green line) and with its slope (red line).

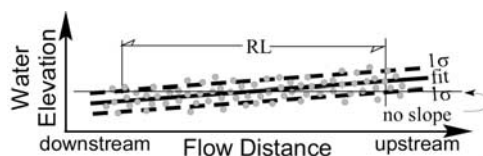
gage location (Figure 6). Resulting slopes are 1.92 cm/km at Manacapuru, 2.86 cm/km at Itapeua, and 3.20 cm/km at Tupe. Errors on the slopes were determined by systematically adjusting polynomial coefficients by the fit error to produce a maximum and minimum estimate of slope. We find errors for Manacapuru to be  $\pm 0.19$  cm/km, for Itapeua to be  $\pm 0.24$  cm/km and for Tupe to be  $\pm 0.34$  cm/km.

[17] Because SRTM collected height data throughout a 10-day acquisition period in February 2000, slope values calculated from the extracted heights include a temporal average. Unlike sudden flood events, the regularity of the Amazon River hydrograph permits SRTM-based slope calculations. For example, using gage data at Manacapuru and Itapeua, the expected change in slope over the 10 days of SRTM mapping is 0.02 cm/km. We assume that this error is incorporated in the slope errors noted above.

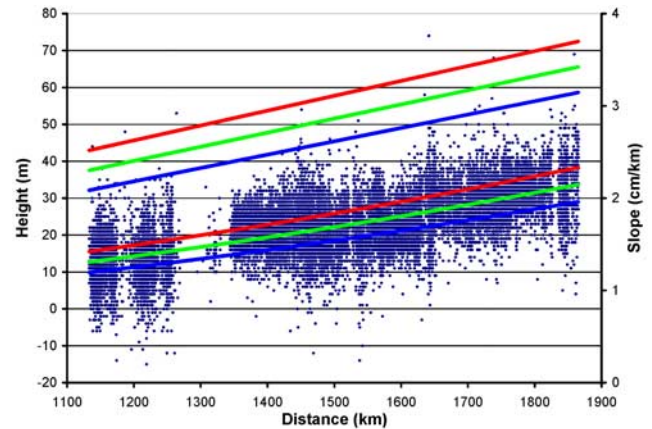
## 6. SRTM-Based Discharge

[18] Discharge is calculated using slopes from the SRTM DEM, widths from the GRFM classification, depths from the nautical charts, and a Manning's  $n$  of 0.025 (Tables 1a–1c). A maximum (minimum) assessment of the error on each discharge value is obtained by using the greatest (least) slope, width, and depth while using the least (greatest)  $n$  value. Resulting discharge values for February 2000 are 84,800  $\text{m}^3/\text{s}$  at Manacapuru, 79,800  $\text{m}^3/\text{s}$  at Itapeua, and 62,900  $\text{m}^3/\text{s}$  at Tupe with errors listed in Tables 1a–1c.

[19] Validation of our SRTM slope-based discharge values is available from gage-based discharge estimates (Tables 1a–1c). At Manacapuru, in-situ discharge



**Figure 5.** Typical data scatter of water surface heights showing minimum reach length needed to accommodate the height error in the data.



**Figure 6.** Polynomial fit to data for 733 km reach length at Itapeua. A 2nd order polynomial was used. Maximum and minimum errors on the polynomial fit and the slope are shown.

averaged over the 10-day, February 2000 mapping period is 90,500  $\text{m}^3/\text{s}$ , only a 6.2% difference. Unfortunately, in-situ discharge measurements are not available for February 2000 at Itapeua or at Tupe. Instead, we use hydrograph comparison methods relying on the year-to-year regularity of the Amazon flood wave. The Itapeua gage-based discharge values are available from 1973 to 1991, and from 1981, a hydrograph period of most similar record to 2000 (i.e., as based on multi-year gage comparisons at Manacapuru and at Manaus). The Itapeua 1973–1991 average February 12–22 discharge is 83,100  $\text{m}^3/\text{s}$  and the 1981 discharge is 74,200  $\text{m}^3/\text{s}$  (4.0% and 7.6% differences, respectively). Tupe does not have an extensive in-situ dataset so it was necessary to compare our values for  $Q$  with those reported by CAMREX over the period 1982–1985 and 1988. Measurements were not continuous but were collected during different seasons and for various positions on the annual flood wave (range 33,200  $\text{m}^3/\text{s}$ –63,100  $\text{m}^3/\text{s}$ , mean 52,700  $\text{m}^3/\text{s}$ ). The maximum annual Tupe discharge occurs in February and March, therefore we use 63,100  $\text{m}^3/\text{s}$  as the representative in-situ value. This is the most subjective comparison of the three locations, but yields essentially no difference (0.3%).

[20] For rivers with large width to depth ratios, equation (3) determines a sensitivity of each parameter in the final derived discharge:

$$Q = (k/n) \left( W Z^{5/3} S^{1/2} \right) \quad (3)$$

where  $W$  is width and  $Z$  is depth [Albertson and Simons, 1964]. From equation (3), if we desire  $\pm 2.5\%$  ( $\pm 5.0\%$ ,  $\pm 10\%$ ) accuracy on  $Q$ , then  $W$  and  $n$  need to be known to  $\pm 2.5\%$  ( $\pm 5.0\%$ ,  $\pm 10\%$ ) accuracy whereas  $Z$  needs a finer measurement accuracy of  $\pm 1.7\%$  ( $\pm 2.6\%$ ,  $\pm 4.0\%$ ) but  $S$  can suffice with an accuracy of  $\pm 6.3\%$  ( $\pm 25\%$ ,  $\pm 100\%$ ). Because the width error is fixed to the remotely sensed pixel size, width accuracies improve with increasing channel width. Slope errors are an intrinsic property of the elevation-measuring instrument. The percent errors on the remotely sensed parameters of  $W$  and  $S$  are between  $-10.5\%$  and  $+8.7\%$ , suggesting that these yield a similar range of error

on derived discharge. The maximum depth error is  $\pm 15.5\%$  whereas Manning's  $n$  errors were fixed at  $\pm 12\%$  for all three study reaches.

## 7. Conclusions

[21] The surprising result of this study is that despite lacking extensive, cross-sectional depth measurements, accurate Amazon mainstem discharge values can be estimated from remotely sensed variables of water-surface slope and channel width coupled with reasonable estimates of Manning's  $n$ . Simple averages from depths reported on nautical charts were sufficient in this study to constrain Manning's velocity equation.

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