Wavelet-transform based edge detection approach to derivation of snowmelt onset, end and duration from satellite passive microwave measurements

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(Received 14 July 2004; in final form 16 June 2005)

We developed a new method for deriving the onset date, end date, duration and spatial extent of snowmelt using satellite passive microwave measurements. Our method exploits the fact that apparent edges are present on the brightness temperature ($T_b$) time series curve corresponding to sharp and abrupt melt-induced transitions of brightness temperature. Through a wavelet transform of daily $T_b$ observations, our method identifies and tracks significant upward and downward edges on the $T_b$ curves. Through variance analysis and bi-modal Gaussian curve fitting, an optimal edge strength threshold is statistically determined to differentiate real snowmelt edges from weak edges caused by noisy perturbations and other non-melt processes. Based on the principle of spatial autocorrelation, a neighbourhood operator is designed to detect and correct possible errors in the melt computations that are purely based on temporal analysis of individual $T_b$ curves. We have implemented the method using C++ programming language and successfully applied it to Special Sensor Microwave/Imager (SSM/I) data collected in 2001–2002 over the Antarctic ice sheet. The computation results were evaluated through visual interpretation of brightness temperature time series and examination of historical near-surface air temperature records.

1. Introduction

Monitoring and measuring snowmelt is important for studying climatic changes in polar regions and for hydrological modelling of snowmelt-runoff in high-latitude and mountainous regions. The polar regions play an important role in the global heat budget by controlling the exchange of heat, moisture and momentum at the surface–atmosphere interface. Wet snow has a relatively low albedo for visible and near-infrared spectral bands and absorbs approximately 45% more incoming solar radiation than high-albedo dry snow (Grenfell et al. 1994). Due to low surface slope, small changes in air temperature can induce large areal changes in wet snow zones in polar ice sheets. Therefore, surface melt of polar ice sheets could serve both as a sensitive indicator of and a strong contributing factor to global climate change (Zwally and Fiegles 1994, Mote and Anderson 1995). In high-latitude and mountainous regions, snow and glacier ice melt can be a significant contributor to the total yearly runoff volume. Accurate and timely measurements of the onset

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and extent of seasonal snow and glacier ice melt are crucial inputs for modelling and forecasting stream discharge, available water resource, and possible floods during spring and summer (Rango 1996, 1997, Schaper et al. 1999, Ramage and Issacks 2003, Ye et al. 2003). Although weather station data or shallow-snow-core records can be used for snowmelt studies, they are often widely scattered. Temporally detailed satellite passive microwave data have been collected by the Nimbus-7 Scanning Multi-channel Microwave Radiometer (SMMR) and the Defense Meteorological Satellite Program (DMSP) Special Sensor Microwave/Imager (SSM/I) sensors. The SMMR recorded microwave radiation every other day from October 1978 to August 1987 in 10 channels (Gloersen and Hardis 1978), including vertical and horizontal polarizations at 6.6, 10.7, 18, 21 and 37 GHz. The SSM/I sensors recorded daily radiation in seven channels, the vertical polarization for 22 GHz and both polarizations for 19, 37 and 85 GHz, spanning from July 1987 to present (Hollinger et al. 1987, Grody and Basist 1997). The National Snow and Ice Data Center (NSIDC) in Boulder, Colorado has processed the SMMR and SSM/I data into a geocoded grid format and made them available to the science community (NSIDC 1992). Many empirical studies (e.g. Mote and Anderson 1995, Abdalati and Steffen 1997, 2001, Joshi et al. 2001) have demonstrated that SMMR and SSM/I data are well suited for acquiring a temporally detailed record of snowmelt occurrences.

In the past decade, various methods have been proposed for extracting snowmelt information from multi-channel satellite passive microwave data. Some methods are based on analysis of brightness temperature values from a single channel, and others use a composite index derived from the passive microwave observations of two channels. Mote et al. (1993) made use of the 19 GHz vertically polarized channel. They calculated the difference between daily brightness temperature \( T_b \) values and the winter mean of brightness temperature values \( T_b - \bar{T}_{\text{winter}} \) to map the snowmelt extent for the Greenland ice sheet. The melt threshold value for the difference index \( T_b - \bar{T}_{\text{winter}} \) was established by comparing passive microwave data and in situ snowpack data. When the brightness temperature \( T_b \) recorded for a pixel on a particular day was in excess of the winter mean plus the difference threshold, that day was designated as experiencing melt. A similar technique has been used by Zwally and Fiegles (1994) to investigate the snowmelt of the Antarctic ice sheet. They calculated the difference between daily brightness temperature \( T_b \) and the mean annual brightness temperature of the 19 GHz horizontally polarized channel \( T_b - \bar{T}_{\text{annual}} \). A threshold value of 30K was selected for the difference index \( T_b - \bar{T}_{\text{annual}} \) to signify melting. Ramage and Isacks (2003) introduced diurnal amplitude variations (DAVs) in their analysis: \( DAV = T_{b,\text{ascending}} - T_{b,\text{descending}} \). DAVs represent the running difference between late-afternoon (usually ascending orbit) and early-morning (usually descending orbit) brightness temperature observations. They combined the threshold of \( DAV (> \pm 10\text{K}) \) and the threshold of \( T_b (>246\text{K}) \) for the 37 GHz vertically polarized channel to determine melt and refreeze timing. Steffen et al. (1993) used a normalized gradient ratio \( (GR) \) between the 19 GHz and 37 GHz horizontally polarized channels as the surface melt index: \( GR = \left( T^{19\text{H}}_b - T^{37\text{H}}_b \right) / \left( T^{19\text{H}}_b + T^{37\text{H}}_b \right) \). They determined a melt threshold for the \( GR \) on the Greenland ice sheet by comparing a time series of \( GR \) values with in situ air temperature data. By replacing the 37 GHz horizontally polarized channel with the 37 GHz vertically polarized channel in the \( GR \), Abdalati and Steffen (1995) coined a cross-polarization gradient ratio \( (XPGGR) \): \( XPGGR = \left( T^{19\text{H}}_b - T^{37\text{V}}_b \right) / \left( T^{19\text{H}}_b + T^{37\text{V}}_b \right) \).
They derived a melt threshold for the \textit{XPGR} to indicate melting by comparing the \textit{XPGR} time series values with \textit{in situ} volumetric water content data.

The aforementioned methods are easy to implement and dependable for differentiating wet snowpack from dry snowpack. However, the estimates for the timing of melt and refreeze are not accurate, because both the single channel indices and composite indices used in these methods are essentially based on the absolute values of brightness temperature. The introduction of \textit{DAVs} makes it possible to differentiate the diurnal melt–refreeze period from the sustained melt period, but improvements to the timing of melt and refreeze are limited. In these methods, a threshold value had to be established no matter which melt index was used. Most researchers used field observations of the volumetric water content of a snow pack or near-surface air temperature from one specific site to derive a threshold value for their selected index. It is known that the depth, grain size, density and vertical stratification of a snowpack vary from location to location. Since a pixel from satellite passive microwave data covers a relatively large ground area, for example 25 km by 25 km for SSM/I data, the \textit{in situ} point observations used to measure snowpack volumetric water content or near surface air temperature may not accurately represent the ground area covered by a pixel. This may result in a biased threshold value. Furthermore, acquiring field observations is very costly and limited by the accessibility of the environment. Consequently, field data are often unavailable for many regions of interest.

Joshi \textit{et al}. (2001) proposed an edge detection method to determine the snowmelt occurrences. They applied a Derivative of Gaussian (\textit{DoG}) edge detector to the 19 GHz vertically polarized channel and determined the timing of melt and freeze-up occurrences. Nevertheless, they utilized a trial-and-error process to determine the edge strength threshold for identifying significant edges corresponding to melt and refreeze events. This process inescapably involved some level of subjectivity and arbitrariness.

We developed a new method to accurately derive melt onset date, melt end date, melt duration and spatial extent from satellite passive microwave data. Our method first decomposes the time series $T_b$ values into multi-scale components through a wavelet transform. Then, the edges are detected at different scales. Variance analysis and bi-modal Gaussian curve fitting techniques are used to statistically determine an optimal threshold for differentiating significant edges corresponding to melting events from weak edges associated with random signal perturbation and noise. Based on the principle of spatial autocorrelation, we also developed a spatial neighbourhood operator for detecting and correcting possible errors brought about by strong noise or the heterogeneity of data pixels. Our method has been implemented using C++ programming language and successfully applied to satellite passive microwave SSM/I data for the Antarctic ice sheet.

2. Wavelet-transform based edge detection method

2.1 Physical basis and methodology overview

Our method is based on the physical mechanism that the microwave emissivity and brightness temperature ($T_b$) of a snowpack increases dramatically in response to the introduction of liquid water content during the snowmelt season. The relationship between the microwave brightness temperature ($T_b$) and near-surface physical temperature ($T_s$) of snow and ice can be represented by the first-order
Rayleigh–Jeans approximation (Foster et al. 1984):

\[ T_b = \varepsilon T_s \]  

(1)

where \( \varepsilon \) is the emissivity of the snow and ice. Although grain size, density, crystal structure and surface conditions (hoar frost, layering or crusts produced by wind or radiation) of snow and ice all contribute to the emissivity of snowpack (Ulaby et al. 1986), changes in the liquid-water content produce the most prominent variations in the emissivity (\( \varepsilon \)) and hence in the brightness temperature (\( T_b \)). A small amount (a few percent by volume) of liquid water induced by the melting process can radically increase the microwave emissivity of a snowpack (Zwally and Gloersen 1977, Ulaby et al. 1986). Therefore, the transition of dry snow to wet snow (liquid water, ice and air) yields a distinct signature: a sharp and abrupt increase in the brightness temperature (\( T_b \)), which is detectable by microwave sensors at frequencies in excess of 10 GHz.

When the brightness temperature (\( T_b \)) time series is plotted as a one-dimensional curve, the structures with an edge (peak) shape are present at melting and refreezing times. Figure 1 shows daily variations in brightness temperature (\( T_b \)) from 1 July 2001 to 30 June 2002 for two passive microwave data pixels in Antarctica, as recorded by the 19 GHz horizontally polarized channel of the passive microwave SSM/I sensor. One is a typical dry pixel, and the other is a typical wet pixel. The dry pixel is located in the interior of the Antarctic ice sheet, where the snowpack remained continuously dry and the brightness temperature varied smoothly. The wet pixel is located on the Antarctic Peninsula, where the snowpack experienced melting during the austral summer. The most prominent feature of the wet pixel is that its brightness temperature (\( T_b \)) increased rapidly at the onset of snowmelt in early summer. This stands in sharp contrast to the lower \( T_b \) observed during non-melt conditions of spring and winter, forming a strong upward step edge (cliff) on the time series curve of daily \( T_b \) observations. An apparent prolonged period (plateau) of elevated brightness temperatures is observed during the summer. During the late summer, a dramatic decrease in brightness temperatures

Figure 1. Daily brightness temperature variations of the 19 GHz channel of SSM/I for typical wet and dry pixels. The wet pixel is located in the Antarctic Peninsula (67.1274°S, 63.7426°W) and the dry pixel is located in the interior Antarctic ice sheet (81.1482°S, 74.7167°W). The horizontal axis represents the sequential number of days during 2001–2002. Day 0 is 1 July 2001.
corresponding to the snow refreezing creates an obvious downward edge (cliff) on the $T_b$ time series curve.

Similarly to Joshi et al. (2001), we exploit the fact that the occurrence of strong and significant edges in the brightness temperature ($T_b$) time series curve signifies snow melting and refreezing events. By detecting and tracking strong and significant edges in the $T_b$ time series curve, we can determine whether and when a pixel experienced melt. The melt onset is the time for the initial presence of liquid water in the upper snowpack, which corresponds to the first significant upward edge on the daily $T_b$ curve. The melt end is the time that the snowmelt ends for the year, which corresponds to the last significant downward edge on the daily $T_b$ curve. The time that elapses between the first significant upward edge and last significant downward edge defines the melt season. The sum of the time lapsed between successive upward and downward edge pairs in the year gives an estimate of the cumulative melt duration, which excludes the time of the intermittent refreezing periods during the melt season.

Our method consists of three key components: edge detection based on multi-scale wavelet transform of daily $T_b$ observations; determination of optimal edge threshold through variance analysis and bimodal Gaussian curve fitting; and a neighbourhood operation for improving the melt computations from pure temporal analysis of $T_b$ time series.

### 2.2 Multi-scale wavelet transforms and edge detection

Different from the Derivative of Gaussian (DoG) edge detector used by Joshi et al. (2001), we adopt a wavelet transform based approach to edge detection. The advantages of this approach lie in the fact that it enables examination of $T_b$ variations at multiple temporal scales. Through multi-scale decomposition of $T_b$ variations, we can analyse the edge strength variations and track snowmelt-induced edges across scales. This is in contrast to the DoG edge detector (Joshi et al. 2001) that identifies edges at a single scale.

We utilize a one-dimensional spline wavelet transform originally developed by Mallat and associates (Mallat and Zhong 1992, Mallat and Hwang 1992, Mallat 1999) to decompose the time series ($T_b$) values. Let $\theta(t)$ be a smoothing, twice differentiable function which approaches 0 at infinity and whose integral is equal to 1. Two mother wavelets, $\psi^1(t)$ and $\psi^2(t)$, are defined as the first and second derivatives of $\theta(t)$ (Mallat and Zhong 1992, Mallat and Hwang 1992):

$$\psi^1(t) = \frac{d\theta(t)}{dt}$$

$$\psi^2(t) = \frac{d^2\theta(t)}{dt^2}$$

$\psi^1(t)$ and $\psi^2(t)$ are the wavelets with compact support because their integral is equal to 0:

$$\int_{-\infty}^{+\infty} \psi^1(t) dt = 0$$

$$\int_{-\infty}^{+\infty} \psi^2(t) dt = 0$$
The dilations of the wavelets $\psi_1(t)$ and $\psi_2(t)$ by a scaling factor ($s$) are denoted as

\[
\psi_s^1(t) = \frac{1}{s} \psi^1\left(\frac{t}{s}\right)
\]

(6)

\[
\psi_s^2(t) = \frac{1}{s} \psi^2\left(\frac{t}{s}\right)
\]

(7)

Let $f(t)$ be a real function to represent the temporal variation of brightness temperature. The wavelet transforms defined with respect to each of these wavelets are given by:

\[
W_1^1 f(s,t) = f \ast \psi_s^1(t) = f \ast \left( s \frac{d \theta_s}{dt}\right)(t) = s \frac{d}{dt} (f \ast \theta_s)(t)
\]

(8)

\[
W_1^2 f(s,t) = f \ast \psi_s^2(t) = f \ast \left( s^2 \frac{d^2 \theta_s}{dt^2}\right)(t) = s^2 \frac{d^2}{dt^2} (f \ast \theta_s)(t)
\]

(9)

where the symbol $\ast$ denotes the convolution operation. From equations (8) and (9), it is clear that the wavelet transforms $W^1 f(s,t)$ and $W^2 f(s,t)$ are respectively the first and second derivative of the brightness temperature signal $f(t)$ smoothed by $\theta_s(t)$ at the scale ($s$). For a fixed scale ($s$), the local extrema of $W^1 f(s,t)$ correspond to zero-crossings of $W^2 f(s,t)$ (the second derivative) and to inflection points of the smoothed signal $f(t) \ast \theta_s(t)$. The position of the local extrema of the wavelet transform $W^1 f(s,t)$ (the first-order derivative) along the time axis indicates the time at which the sharp variations occur, and the magnitude of the wavelet transform modulus $|W^1 f(s,t)|$ at the corresponding locations indicates the strength of the edges caused by sharp transitions. The zero-crossings of $W^2 f(s,t)$ (the second derivative) give position information for the extrema, but it is difficult to differentiate the maxima from the minima with $W^2 f(s,t)$. Since we need to track and differentiate the upward edges from downward edges, the wavelet $W^1 f(s,t)$ is preferred in our analysis.

Through the wavelet transform, the passive microwave brightness temperature signals ($T_b$) are decomposed into multi-scale components that are well localized in both time and frequency. The wavelet transform works as a mathematical microscope that can focus on a specific part of the signal to extract local structure and singularities. Irregular edge structures on the $T_b$ time series carry essential information for inferring the underlying snow melting and freezing processes. With multi-scale edge detection, we can identify and characterize the local maxima of a wavelet transform modulus $|W^1 f(s,t)|$ at a range of scales. The scale defines the size of the neighbourhood that is used to compute the brightness temperature changes. Edges at the scale ($s$) are defined as local sharp variation points of $f(t)$ smoothed by $\theta_s(t)$. By increasing the scale ($s$), the convolution with $\theta_s(t)$ removes small signal fluctuations and data noise and highlights sharp variations of large structures. By tracking the evolution of the local maxima of the wavelet transform modulus $|W^1 f(s,t)|$ across scales, we can determine the timing and type of sharp brightness temperature transitions (singularities or discontinuities) indicated by irregular edge structures.

To process brightness temperature ($T_b$) time series recorded by satellite passive microwave sensors, we implemented a fast discrete wavelet transform algorithm.
(Mallat and Zhong 1992). For the fast discrete numerical computation, the wavelet transform is limited to dyadic scales: $2^j$ ($j=1, 2, \ldots, n$). From the discrete wavelet transform at each scale, we can detect the modulus maxima by finding the points where $|W^j f(2^j t)|$ is larger than or equal to its two closest neighbour values and strictly larger than at least one of them. We record the abscissa ($t$) and the value of $W^j f(2^j t)$ for each of the local extrema. The values of $|W^j f(2^j t)|$ at the extrema locations measure the derivative at the inflection points, namely, the magnitude of edge strength.

Since 365 days of brightness temperature values are available for a year, it is possible to decompose the $T_b$ time series up to the scale $2^8$. However, our extensive experiments show that it is sufficient to decompose the $T_b$ time series up to the scale $2^4$, because the wavelet components at the scale larger than $2^4$ contain only the information regarding the general seasonal trend of brightness temperature variations. The melt and refreeze signals are contained in the decomposed wavelet components at the scale less than $2^4$. Figure 2 shows a numerical example to illustrate the wavelet transform and multi-scale edge detection process. Figure 2(a) is the daily brightness temperature ($T_b$) time series for the year. Figure 2(b) is a multi-scale representation of the original ($T_b$) signal from the wavelet transform. Each Dirac delta function in figure 2(c) shows the position (timing) of a modulus maxima, and its height (value of $|W^j f(2^j t)|$) indicates the edge strength at the corresponding location. As shown in figure 2(b), at the finest scale ($2^1$) the wavelet transform $W^j f(2^1 t)$ is highly influenced by noise. The number of extrema created by white noise decreases as the scale increases from the scale $2^1$ to the coarser scales (figure 2(c)). The modulus extrema that are created by sharp transitions of snow melting and refreezing are persistent and propagate from fine scales into coarser scales. Their modulus amplitude decreases only slightly when the scale increases. In other words, sharp transitions create sequences of extrema that converge towards the corresponding location at fine scales. We select the modulus maxima that propagate at least up to the scale $2^3$ for our snowmelt analysis. The original brightness temperature signal can be reconstructed by the inverse wavelet transform. $S^j f(2^3 t)$ is the reconstructed signal from the inverse transform of the wavelet components at scales equal to or larger than $2^4$. As shown in figure 2(b), $S^j f(2^3 t)$ represents large-scale seasonal variations of the brightness temperature.

2.3 Determination of the optimal edge strength threshold

By tracking the local extrema of wavelet transform modulus across scales, we can recognize the perturbations caused by data noise and focus on the strong brightness temperature edges created by some geophysical processes. Besides melting and freezing, other processes such as near-surface air temperature variations below the melting point, katabatic winds and new snowfall may also cause significant brightness temperature transitions that may survive up to the specified scale along with the melt- and freeze-induced sharp transitions.

We developed a procedure to differentiate the sharp transition induced by melting and refreezing processes from other transitions caused by non-melt processes. First, we identify a critical value as a dividing point to classify the local extrema of wavelet transform modulus into two groups: a lower maxima group and an upper maxima group. The critical value for a pixel is defined as the group dividing point that
maximizes the variance ratio ($VR$):

$$VR = \frac{VAR_{between}}{VAR_{within}}$$  (10)

Figure 2. Multi-scale decomposition of $T_b$ time series by a fast discrete wavelet transform. (a) Original daily brightness temperature; (b) wavelet transform of the original brightness temperature at eight scales and reconstructed large-scale seasonal variation from the wavelet components at scales equal to or larger than $2^4$; and (c) wavelet transform modulus extrema detected at the first four scales.
where $VAR_{between}$ is the variance of local extrema between the lower group and upper group, and $VAR_{within}$ is the sum of the variance within the lower group and the variance within the upper group. With such a defined critical value, we aim to maximize the difference between the lower and upper groups and to minimize the differences within each group.

Figure 3 illustrates the determination of a critical value for classifying local extrema of wavelet transform modulus into lower and upper groups. The modulus extrema are detected from the wavelet transform as described above (figure 3(a)). As shown in figure 3(b), the variance ratio reaches its maximum when the value of 49.5 is selected as the dividing point to classify these detected modulus extrema into lower and upper groups. Therefore, 49.5 is the critical value for this pixel. The modulus extrema with a magnitude equal to or above 49.5 are identified as significant edges in the upper group.

Figure 3. Variance analysis for determining the critical value. (a) The wavelet modulus extrema and significant upward and downward edges identified with the critical value; (b) variation of variance ratio ($VR$) with different dividing points.
We use the above procedure to derive a critical value for every pixel. By sampling both wet pixels that experienced melting and dry pixels that remained dry, we analysed the statistical characteristics of critical values of wet and dry pixels. Figure 4 shows a histogram of critical values for a sample of pixels. The histogram exhibits an obvious bi-modal shape, namely, two dominant modes (lobes) and a broad valley. The mode with a low mean value indicates a cluster of dry pixels, while the mode with a higher mean represents a cluster of wet pixels. Located in the valley of the histogram are pixels that partially experienced melting. Namely, only a portion of the pixel rather than the entire pixel went through the melting process. We computationally derived an optimal threshold value for separating wet pixels from dry pixels. The optimal threshold value corresponds to the histogram minimum (the valley point).

To model a bi-modal histogram, we used a mixture of two Gaussian (normal) distribution functions. Each component Gaussian distribution can be statistically defined by two parameters: mean and standard deviation. The probability density function \( p(x) \) of the mixed bi-modal histogram can be modelled by equation (11) with five unknown parameters (Chow and Kaneko 1972, Gonzalez and Wintz 1987):

\[
p(x) = \frac{p_1}{\sqrt{2\pi}\sigma_1} \exp \left[ -\frac{(x-\mu_1)^2}{2\sigma_1^2} \right] + \frac{1-p_1}{\sqrt{2\pi}\sigma_2} \exp \left[ -\frac{(x-\mu_2)^2}{2\sigma_2^2} \right]
\]

where \( \mu_1 \) and \( \mu_2 \) are the mean values of two component normal distributions, \( \sigma_1 \) and \( \sigma_2 \) are the standard deviations about the means, and \( p_1 \) is the percentage of dry pixels in the sample. We iteratively fitted the five parameters for the non-linear bimodal Gaussian curve using the Levenberg–Marquardt method (Press et al. 1992). For the given bi-modal histogram in figure 4, the optimally fitted parameters are: \( \mu_1 = 6.35, \sigma_1 = 1.26, \mu_2 = 30.66, \sigma_2 = 7.4 \) and \( p_1 = 0.277 \). Apparently, the dry pixels are more homogeneous, as indicated by a small standard deviation of 1.26, while the wet pixels are more heterogeneous with a much larger standard deviation of 7.4. Overall, the fitted bi-modal Gaussian curve closely matches the histogram.

Based on the five Gaussian parameters, an optimal threshold value \( T \) (the valley point in the histogram) can be analytically computed. The optimal threshold value \( T \) is so defined that all pixels with a critical value below \( T \) are considered dry pixels.
and all pixels with a critical value above $T$ are considered wet pixels. The probability of erroneously classifying a wet pixel as a dry pixel is (Gonzalez and Wintz 1987):

$$E_1(T) = \int_{-\infty}^{T} \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right] dx$$

Similarly, the probability of erroneously classifying a dry pixel as a wet pixel is:

$$E_2(T) = \int_{T}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right] dx$$

The overall probability of the classification error is:

$$E(T) = (1-p_1)E_1(T) + p_1E_2(T)$$

By minimizing the probability of misclassification $E(T)$ through Liebnitz’s rule, we can derive the optimal threshold ($T$):

$$T = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$A = \sigma_1^2 - \sigma_2^2$$
$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$
$$C = \sigma_1^4\mu_2^2 - \sigma_2^2\mu_1^2 + 2\sigma_1^2\sigma_2^2\ln\frac{\sigma_2p_1}{\sigma_1(1-p_1)}$$

One of two possible solutions for $T$ can trivially be dropped by checking the condition: $\mu_1 < T < \mu_2$. Using the equation (15), the optimal threshold value ($T$) for the histogram in figure 4 is determined to be 10.8.

The critical value determined through variance analysis for each pixel is first compared with the optimal edge threshold ($T$). The pixels with a critical value less than $T$ are designated as dry pixels and no further processing is conducted for those pixels. The pixels with a critical value in excess of the threshold value $T$ are marked as wet pixels. For each wet pixel, the melt onset date, melt end date and melt duration are determined by tracking its wavelet transform modulus extrema (edges) that survive up to scale $2^{3}$. The modulus maxima with a value of $|W^3f(2^3,t)|$ larger than its critical value are considered to be melting-induced upward edges, and the modulus minima with an absolute value of $|W^3f(2^3,t)|$ above its critical value are considered to be refreeze-induced downward edges. Other extrema with an absolute value of $|W^3f(2^3,t)|$ below its critical value are considered insignificant edges induced by non-melt processes. By locating the position of the first significant modulus maximum (upward edge) whose $|W^3f(2^3,t)|$ exceeds its critical value, we are able to identify the melt onset date for the pixel. Then, we identify the first modulus minimum (downward edge) that follows the first modulus maximum and whose $|W^3f(2^3,t)|$ exceeds its critical value, and calculate the time interval for the first melt–refreeze period between the first pair of modulus maximum and minimum. By repeating this process, we calculate the time intervals for the remaining melt–refreeze periods. By adding the time intervals for all identified melt–refreeze periods for the year, we obtain the total melt days in the year, namely melt duration. The
last modulus minimum (downward edge) whose $|W^j(f(2^3,t))|$ is larger than the critical value indicates the end date of the melt season for the year. For the wet pixel shown in figure 3, the computation results are: the melt onset date is 28 November, the melt end date is 14 February, and the melt duration (total melt days) is 73 days, 5 days shorter than the melt season (78 days).

2.4 Enhancement of melt computation by neighbourhood operation

The wavelet transform and the multi-scale edge detection processes described above sequentially analyse the $T_b$ time series pixel by pixel, in which each pixel is treated as an independent entity. Essentially, the melt onset date, melt end date and melt duration are computed from the temporal analysis of daily brightness temperature variation for each individual pixel, and the spatial relationship between pixels is not taken into account during the computation.

It is well known that snow melt and freeze phenomena have strong spatial autocorrelation. The spatial autocorrelation (dependence) arises due to the geographical continuity of the fundamental factors controlling melting and freezing. In general, near-surface air temperature, snowpack physical temperature, surface topography, dielectric properties of snowpack and other factors vary gradually and smoothly over space. Therefore, geographically adjacent pixels within a neighbourhood are expected to have a similar melt onset date, melt end date and melt duration.

Nevertheless, the daily $T_b$ time series may be contaminated and distorted in some conditions, such as strong noise, sensor malfunction, thick clouds, geolocation errors and the existence of ponding water or strong relief within a data pixel. The contaminated data can mislead the wavelet-transform based edge detection analysis and result in erroneous outputs for melt onset date, melt end date or melt duration. In addition, the computation results from the wavelet-transform based edge detection may be sensitive to the critical value, and a slight overestimate or underestimate of the critical values may cause incorrect outputs. We observed that these errors are spatially random and often come into sight as isolated wet pixels surrounded by dry pixels, or isolated dry pixels surrounded by wet pixels, or wet pixels whose melt onset date, melt end date or melt duration are substantially different from those of their neighbouring pixels. While the presence of such a local ‘salt and pepper’ melting pattern cannot be completely excluded a priori given the poor spatial resolution of passive microwave data (25 km), it can be at least regarded as a flag that signifies a possible error.

Based on the spatial autocorrelation principle, we designed a median difference operator to automatically check and flag potential errors. Each pixel was examined with a $3 \times 3$ neighbourhood (window). The difference between the melt onset date of the pixel under examination and the median of the melt onset dates of its eight neighbours is calculated. Similarly, the median differences for the melt end date and melt duration are also computed for each pixel. If one of these three median differences for the pixel is larger than a specified value, we mark this pixel as a potential error. For a marked pixel, we relax its critical value stepwise by increments of $\pm 0.5$ at a time and re-compute its melt onset date, melt end date and melt duration. For a pixel with too early an onset date, too late an end date or too long a duration compared with the median of its neighbouring pixels, we decrease its critical value. For a pixel with too late an onset date, too early an end date or too short a duration, we increase the critical value. If the newly computed melt onset
date, melt end date and melt duration are within the specified range, they are accepted as correct results. If the newly computed results are still out of the specified range, we simply pop up an interactive graphical interface to plot the brightness temperature curve and determine the melt onset date, melt end date and melt duration through human visual interpretation of the temperature curve in relation to that pixel’s neighbouring pixels and its geographical location.

Figure 5 illustrates the neighbourhood operation for correcting a potential error. The original result for melt onset date is shown in figure 5(a). The neighbourhood operation detects a potential error since the melt onset date of the central pixel is 58 days earlier than the median of its neighbours. With an increased critical value, a new melt onset date is calculated for this pixel as shown in figure 5(c). The new result is consistent with that of the neighbouring pixels (figure 5(b)). By incorporating the spatial context with a neighbourhood operation, we are able to detect and correct potential errors and boost the calculation results from pure temporal analysis of $T_b$ time series.

Figure 5. Error detection and correction using a neighbourhood operation. (a) Melt onset date before neighbourhood operation; (b) melt onset date after neighbourhood operation; (c) relaxed critical value compared with the original critical value.
3. Application example and numerical results

Our method has been implemented using C++ programming language. It consists of several routines with the following functions: extracting a specified channel from the SSM/I EASE grids to form \( T_b \) time series, performing the fast discrete wavelet transform, identifying wavelet transform modulus extrema for edge detection, determining the critical value and edge strength threshold, and boosting the melt computation results with a neighbourhood operation. Here we show the computation results for the Antarctic ice sheet during the 2001–2002 melt season to demonstrate the performance of our new method. With our method, we have successfully processed the SMMR and SSM/I passive microwave data for the years 1978–2004. In a subsequent paper we will report the spatio-temporal variability of snowmelt in the Antarctic ice sheet over the past two and a half decades.

The satellite passive microwave data used in this example are the SSM/I data from 1 July 2001 to 30 June 2002, which were provided by the National Snow and Ice Data Center (NSIDC) in Level 3 EASE-Grid format. The ESAE-Grid SSM/I data have two separate files for ascending and descending orbit observations respectively. The ascending orbit observations during 2001–2002 were made in the late afternoon (about 1800 hours local time), and the descending orbit observations were made in the early morning (about 0600 hours local time). The SSM/I data have seven different channels. Our experiments show that although all channels show similar responses to melt events, the 19 GHz horizontally polarized channel exhibits the strongest melt-induced edges on the \( T_b \) time series. Therefore, we selected the ascending orbit observations of this channel for our snowmelt analysis. The nominal spatial resolution of this channel is 25 km, and a grid of 182 \( \times \) 222 pixels covers the entire Antarctic continent. A coastline mask (Liu and Jezek 2004) was applied in order to extract ice sheet pixels. The analysis was carried out only for pixels that lie on the ice sheet in order to eliminate ocean contamination.

For every ice pixel, we form 365 days of \( T_b \) time series starting with 1 July as the first day in order to centre the Antarctic austral summer season in the middle of the \( T_b \) curve. For each pixel, we compute the wavelet transform up to the scale \( 2^4 \). Then, we identify the wavelet transform modulus extrema that survive up to scale \( 2^3 \). Based on the values of the identified modulus extrema, a critical value is computed for each pixel by maximizing the variance ratio. By fitting a bi-modal Gaussian curve on the critical value histogram of a sample of 3896 pixels over 25 years, an optimal edge threshold is computed to be 10.8. Pixels with a critical value above 10.8 are labelled as wet pixels. For a wet pixel, we computed its melt onset date, melt end date and melt duration by tracking its upward and downward edges as described above.

The spatial combination of the above computation results produces three grids with 182 \( \times \) 222 pixels: the melt onset date grid, the melt end date grid and the melt duration grid. A 3 \( \times \) 3 median difference operator as described earlier is applied to these three grids. If the median difference for the melt onset, melt end or melt duration exceeds 2 weeks (14 days), the pixel is marked as a potential error. Its melt onset date, melt end date and melt duration are recomputed with a relaxed critical value or re-estimated by visual interpretation with a graphical interface support as explained above.

Over most of the Antarctic ice sheet, surface temperatures remain below freezing throughout the year. Figure 6 shows the brightness temperature on the peak melt day during the austral summer from 2001 to 2002. The occurrence of snowmelt in
the Antarctic ice sheet is confined to a narrow strip near the coast. The spatial variations of the melt onset date, melt end date and melt duration during 2001–2002 over the Antarctic ice sheet are shown respectively in figures 7, 8 and 9. The total area classified as wet pixels with at least 1 day of melting is 1 127 500 km$^2$, covering 8.2% of the Antarctic continent. Relatively extensive surface melting occurred only on the Larsen Ice Shelf in the Antarctic Peninsula, the Wilkins Ice Shelf and George VI Ice Shelf around Alexander Island, the Amery Ice Shelf, the Shackleton Ice Shelf, the West Ice Shelf, and ice shelves along the coast of Queen Maud Land. Scattered, small-scale snowmelt regions are detected along the coastal margin of Marie Byrd Land and Wilkes Land. In addition, a brief melting is also detected on the rim of the Ronne Ice Shelf, Filchner Ice Shelf, and over the Ice Streams in the West Antarctica, but no melting is observed on the Ross Ice Shelf.

The earliest snowmelt in Antarctica occurred on the northerly ice shelves with relatively low latitude, including the Larsen Ice Shelf, the Wilkins Ice Shelf, the George VI Ice Shelf, the West Ice Shelf and the Shackleton Ice Shelf (figure 7). The latest melting in the year took place in the Larsen Ice Shelf, the Wilkins Ice Shelf, the George VI Ice Shelf and the Abbot Ice Shelf, as shown in figure 8. Overall, the Larsen Ice Shelf, the Wilkins Ice Shelf, the George VI Ice Shelf, the West Ice Shelf and the Shackleton Ice Shelf experienced the most intensive melting in Antarctica (figure 9). About 14.4% of the total melt regions experienced less than 1 week of melting. About 85% of the melt regions experienced 7–90 days of melting, and only 0.7% of the melt regions experienced over 90 days of melting (figure 10). As expected, a general spatial pattern is: with increasing elevation and increasing distance to the coastline, the melt onset date was later, the melt end date was earlier
and the melt duration was shorter. Spatial autocorrelation is evident, and geographically adjacent pixels have similar melt onset, melt end date and melt duration.

Since the melt–refreeze periods for each pixel are tracked and recorded in our method, we are able to show the daily snowmelt extent during the summer season. Melt begins to occur in early November in a very limited area of the Antarctic Peninsula, but until the end of November only 3.35% of the melt regions experienced melting. Starting in early December, the melting coverage increases rapidly. The greatest melt extent of 888 125 km$^2$ occurred on 11 January 2002, reaching 79.3% of the total melt regions. The melt coverage decreases from early February, and 94.2% of the melt regions terminated their melt season before March (figure 11). Figure 12 demonstrates the advance and subsequent retreat of the ‘melt wave’ as the melt season progresses from December to February. Obviously, the melt wave advanced approximately from coastal margins to inland and from lower elevations to higher elevations.

Our automated method is evaluated and validated in two ways. First, the melt onset date, melt end date and melt duration days computed by our automated method are compared with the visual interpretation of $T_b$ time series curves for 50 wet pixels, which are randomly selected in the detected melt regions. In most cases, our algorithm-retrieved melt onset date, melt end date and melt duration are

![Figure 7. Spatial variation of snowmelt onset date over the Antarctic continent during 2001–2002. White areas indicate no melting.](image-url)
identical to the results recognized by human visual interpretation. The overall root mean squared error (RMSE) of our method is 2.3 days for melt onset date, 1.6 days for melt end date and 3.7 days for melt duration. For these 50 wet pixels, the RMSE of the conventional melt index method by Zwally and Fiegles (1994) is 6.8 days for melt onset date, 5.9 days for melt end date and 6.4 days for melt duration. As shown in figure 13, the use of annual mean temperature plus 30K as the melt threshold in Zwally and Fiegles (1994) tends to produce too late a melt onset date, too early a refreezing date for each melt event, and hence too short a melt duration, compared with visual interpretation results. Secondly, the $T_b$ time series curves from the SSM/I data are also compared with coincident near-surface air temperature data at several automated weather stations. The relationship between the detected melting periods and the occurrence of near surface air temperatures above and near the melting point is examined. One example is shown in figure 14. The algorithm-retrieved snow melt onset date, melt end date and melt duration generally correspond to the occurrence of the air temperature above or near the melting point (0°C). Nevertheless, it should be pointed out that the weather station recorded an above 0°C temperature on 30 September 2001 which was not reflected by the passive microwave brightness temperature. The possible reason is that the point measurement of air temperature on that day may represent only a small area around the weather station, and the majority of the ground area covered by the

Figure 8. Spatial variation of snowmelt end date over the Antarctic continent during 2001–2002.
SSM/I pixel (25 km x 25 km) still had air temperature below 0°C and did not experience melting on that day.

4. Discussion and conclusions

With the ability to acquire data in all weather conditions and operate day and night, temporally detailed records for snowpack have been established globally by satellite passive microwave SMMR and SSM/I sensors since 1978. Due to their great sensitivity to changes in snowpack liquid-water content, multi-channel passive microwave data provide a practical means for scientific investigation of melt occurrences at a large geographical scale. Based on the wavelet transform, we developed a novel method for deriving snowmelt information from satellite passive microwave data. We have demonstrated that our method not only delineates melt extent but also accurately determines the melt onset date, melt end date and melt duration.

Our method exploits the fact that the brightness temperature has sharp and abrupt transitions for a snowpack that experiences melt. Through analysis of the evolution of wavelet transform modulus extrema across scales, we are able to separate the effects of data noise and non-melt-induced fluctuations and focus on significant edges induced by melt and freeze events. Compared with the traditional...
methods that use a melt index with an absolute threshold, our method provides more accurate estimates for melt onset date, melt end date and melt duration. This is because our method explicitly searches and tracks sharp brightness temperature transitions to obtain the precise timing of melt and freeze occurrences.

Another attractive feature of our method is that an optimal edge threshold is statistically determined using variance analysis and a bi-modal Gaussian distribution model fitted on a sample of wet and dry snow pixels. This circumvents the limitations and frequent unavailability of field observations for threshold determination, and also overcomes the arbitrariness and subjectivity of the trial-and-error approach used in previous studies. Spatial context and autocorrelation have been ignored in previous snowmelt algorithms. Our method explicitly incorporates the spatial context for the first time through a median difference operator to detect and correct potential errors from pure temporal analysis of brightness temperature time series. The spatial contextual reasoning and operation effectively enhances the reliability and accuracy of snowmelt computation.

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Figure 10. Frequency distribution of snowmelt duration for wet pixels.

Figure 11. Temporal variation of melt extent during 2001–2002.
The main disadvantage of satellite passive microwave data is the coarse spatial resolution. Even though the SSM/I data were resampled into 25 km by 25 km pixels for channels at 19 GHz, their natural resolution is 70 km along-track and 45 km

Figure 12. Advance and retreat of the melt wave over the Antarctic continent.

Figure 13. Comparison of melt analysis results between the wavelet-transform based method, conventional melt index threshold method and visual interpretation.
cross-track. The brightness temperature of each data pixel represents the average snow condition over approximately 3150 km². With the optimal threshold, we minimized the misclassification between dry pixels and wet pixels. Nevertheless, it should be noted that even if the melt extent within a pixel is as high as 1400 km², melt occurrence may not be detected because the melt portion is smaller than 50% of the ground area of one pixel. Furthermore, no direct information about the actual ablation amount or percentage of melting can be derived for each data pixel with passive microwave data.

Our method was tested and evaluated for the Antarctic ice sheet. We believe that it will be equally applicable to other snow- and ice-covered areas. Given the historical accumulation and continual acquisition of passive microwave data, our method can be used to derive long-term records of the timing, temporal frequency and spatial extent of snowmelt occurrences for analysing regional and global climate change.

Acknowledgement
This work was supported by the NASA grant NAG5-10112 and the NSF grant No. 0126149. The authors wish to thank the National Snow and Ice Data Center (NSIDC) in Boulder, Colorado for providing the SSM/I EASE-Grid brightness temperature data for this research project.

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