INTRODUCTION-

The interpretation of the climatic record stored in the great polar ice sheets is one of the central problems in glaciology. Almost all of the climatic data from these ice sheets has come from the study of ice core stratigraphy. Other physical parameters such as the distribution of temperature reflect the ice sheet's integrated response to changes in its environment, and so can provide another approaching to reconstructing past climates.

The usefulness of the temperature distribution for inferring past climate changes depends mainly on the fact that the time needed for the ice sheet to respond fully to such changes (the ice sheet 'memory') may be quite long. (By climate here we restrict ourselves to mean surface temperature and accumulation rate, as these are the parameters that have the greatest effect on the dynamical state of the ice sheet.) For example, Whillans [1981] calculates that the East Antarctic ice sheet takes 30,000 to 35,000 years to equilibrate after a step change in accumulation rate, and even longer for a step change in surface temperature. The ice sheet then may be just beginning to adjust to the warming that occurred at the end of the Wisconsinan glacial period.

Proposed models for the East Antarctic Wisconsinan-Holocene climatic transition (that is, surface temperature and accumulation rate specified as a function of time) can be tested using the measured temperature distribution in the ice sheet. One can solve the heat equation with the appropriate time-dependent boundary conditions and generate a temperature profile for a given site that can be compared with the measured profile. In essence, this approach attempts to solve the heat equation in reverse, that is, to derive the boundary conditions that produce a particular temperature distribution. Generally this problem does not have a unique solution, but this approach does serve to eliminate from consideration the proposed climatic schemes that result in poor agreement between calculated and measured profiles. If other information is available that serves to reduce the number of potential climatic change models, then it may be possible to uniquely specify a climatic scenario for a given site. This data may come from the stable isotope record, secular variations in dust and microparticles, and the chemical stratigraphy measured in ice cores from the site; or even a historical record. (See Jenssen and Campbell [1983] and Budd and Young [1983, 1983a] for a more complete general discussion of this ap-
proach and its application to the Camp Century, Byrd Station, and Vostok temperature profiles.)

For example, Johnsen [1977] modeled shallow (50-100 m) temperature profiles at Crete and at Dye 3 on the Greenland ice sheet by using oxygen isotope and historical data to specify the surface temperature boundary condition. At both sites he found good agreement between calculated and measured profiles (0.02 °C), and concluded that recent temperatures on the south Greenland Dome have been similar to west coast rather than east coast Greenland temperatures. Johnsen's work also seems to show that the oxygen isotope record at Crete, at least on a decades time scale, contains a good proxy record of surface temperature variations.

For deep temperature profiles, a major problem with this type of modeling is that the velocity distribution through the ice sheet must be known before calculated temperature distributions can be considered to realistically reflect the integrated effects of secular changes in boundary conditions. This is because in an actively deforming medium heat is transported by advection as well as conduction, and this advective transport can significantly affect the rate at which the temperature distribution within the ice sheet adjusts to surface changes. While the variation of horizontal velocity with depth at a given site can be inferred from borehole tilting measurements, the vertical velocity profile is much more difficult to measure, and may be the larger velocity component at sites close to the local ice divide.

The fact that the temperature distribution may depend critically on the velocity profile as well as the boundary conditions provides a way to determine the ice velocity at a given site if the boundary conditions there can be specified with some confidence. One then generates temperature profiles resulting from a number of assumed velocity profiles, and selects the model which gives the best agreement with the measured temperature distribution. One problem with this method is that more than one flow model may generate good-fit profiles; another is that it is virtually impossible to consider all flow models that satisfy the appropriate dynamical boundary conditions. In spite of these limitations, this approach can serve to suggest the nature of the ice flow at sites where little is known about the ice dynamics.

The site of interest here is Dome C on the East Antarctic plateau (figure 1). There a 905 m ice core was recovered during the 1977-78 field season
Figure 1.
Antarctic location map. Dome C coordinates are latitude 74°39' S, longitude 124°10' E, and elevation 3215 m.
[Lorius and Donnou, 1978]. Subsequent analysis based on a simple flow model has shown that the available climatic record spans the Wisconsinan-Holocene glacial-interglacial transition, and seems to extend back to about 30,000 y.b.p. [Lorius and others, 1979; Lorius and others, 1984]. A detailed 800 m temperature profile [Ritz, et al., 1982; Gillet and Rado, 1979] also has been measured, and is shown in figure 2. The Dome C camp is located at or very near an ice divide, where the horizontal ice movement vanishes. The climate on the East Antarctic plateau is very cold and arid; at Dome C, the mean annual temperature is about -53.7 °C, and the current accumulation rate is about 0.040 m a\(^{-1}\) of ice [Petit et al., 1982; Palais et al., 1982], corresponding to about 10 cm of snow per year. Almost all of this precipitation seems to occur in the winter; over three austral summers we have observed only clear-sky precipitation resulting in no net accumulation. During the winter the camp is closed and the relative amounts of snowfall due to clear-sky precipitation and synoptic storms are not known. The Dome C site is characterized by light winds of variable direction, and by strongly diurnal insolation during the summer, with daily maximum and minimum temperatures differing by about 20 °C.

The Dome C borehole being located at an ice divide complicates the problem of specifying the velocity profile. This is because shear stresses vanish at the divide and conventional flow models which ignore the longitudinal deviator stresses are not appropriate. One could attempt to solve for the stress distribution at the ice divide as a boundary value problem, and then calculate the longitudinal strain rate from a flow law. Measurements at Dome C show that temperature, crystal orientation and size, and dust and impurity content vary greatly with depth in the upper 900 m. These parameters seem to have significant, but poorly understood effects on the ice rheology, and they would have to be included in any attempt to model the ice flow realistically.

Here, instead, we attempt to determine the general functional dependence of the vertical velocity at the Dome C ice divide (or, alternatively, the longitudinal strain rate) by specifying the surface and basal boundary conditions and then considering various forms for the velocity profile, using the general approach discussed above. Specifically, we assume two general forms for the depth variation of the longitudinal strain rate. Each of these two models comprises a set of simple, monotonic polynomials of degree p and can be characterized by that single parameter.
Figure 2.

Dome C temperature profile. Values are taken from Ritz, [1982]. Values in the upper 400 m were measured by thermistors E and C, which differed typically by about 0.01 °C. The averaged values are plotted as open circles. Points plotted as solid circles were measured with thermistor A, which differed from E and C by about 0.04 °C. The solid line is a plot of measured values normalized to the thermistor A data set, smoothed by a low-pass filter.
THE ICE SHEET MODEL -

In a moving medium, the heat equation takes the form:

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \frac{1}{c} \nabla (c k \nabla \theta) + Q/c \tag{1}$$

where $\theta$ is the temperature, $c$ is the volumetric heat capacity, $Q$ is the rate of internal heating per unit volume and $k$ is the thermal diffusivity.

The Dome C camp is at, or at least very near, an ice divide, and so we assume that horizontal velocities ($v_x$ and $v_y$) are much smaller that the vertical velocity and can be ignored. Ignoring horizontal gradients in temperature further reduces the heat equation to the one-dimensional form:

$$\frac{\partial \theta}{\partial t} + [v_z(z) - (1/c)\partial (c k) / \partial z] \frac{\partial \theta}{\partial z} = k a^2 \partial^2 \theta / \partial z^2 + Q/c \tag{2}$$

where $z$ is positive downward from the surface. We have implicitly assumed that the convective transfer of heat is unimportant. Although part of the measured temperature profile is above the pore close-off zone at about 100 m, studies by Dalrymple et al. [1966] at the South Pole and Weller and Schwerdtfeger [1977] at Plateau Station indicate that the non-conductive transfer of heat in polar firn seems to be negligible below about 8 m depth.

The volumetric heat capacity, $c(z, \rho)$, is defined as the product of density and the specific heat of ice, and here we take:

$$c(\rho, z) = (2.096 \times 10^3)(1 + 0.00370) \rho(z) \quad J \cdot ^\circ C^{-1} \cdot m^{-3} \tag{3}$$

where $\rho(z)$ is the density in kg m$^{-3}$. The variation of density with depth in the upper 50 m at Dome C can be described by the function

$$\rho(z) = 922 - 564 \exp(-0.017 z) \quad \text{kg m}^{-3} \tag{4}$$

with a correlation coefficient of .97 [Alley et al., 1982], and is in good agreement with densities measured down to 200 m depth (D. Raynaud, personal communication). The weak temperature dependence of the specific heat of ice as measured by Giauque and Stout [1936] has also been included in equation (3).
The rate of internal heating, $Q$, here consists of two components. One is the heat generated by deformation in the fully densified ice:

$$Q_{\text{def}} = \dot{\varepsilon}_{ij} \tau_{ij} \quad \text{J} \ a^{-1} \ m^{-3}$$

(5)

where $\tau_{ij}$ and $\dot{\varepsilon}_{ij}$ are the components of the stress deviator and strain rate tensor respectively. Because the surface slope at an ice divide is zero, the contribution of shear can be ignored. From mass continuity, the average longitudinal strain rate is:

$$\dot{\varepsilon} = \dot{\alpha} / H = 10^{-5} \ a^{-1}$$

(6)

where $H$ is the ice thickness in m and $\dot{\alpha}$ is the annual accumulation rate in m a$^{-1}$. From equation (5), we can estimate the resulting rate of temperature increase to be:

$$Q_{\text{def}} / c = 2 \dot{\varepsilon} \sigma / c = 10^{-9} \sigma \quad ^{\circ}C \ a^{-1}$$

(7)

with $\sigma$ the longitudinal deviator stress in bars. We see that for typical deviator stresses on the order of one bar, the contribution of deformational heating at Dome C is negligible. Heat is also generated as the firn densifies and the rate of heating is:

$$Q_{\text{den}} = \dot{\alpha} g \left( \rho_i / \rho(z) \right)^2 \ (\partial \rho / \partial z) \int \rho(z) \ dz \quad \text{J} \ a^{-1} \ m^{-3}$$

(8)

where $g$ is the gravitational acceleration, 9.8 m/s$^2$, and $\rho_i$ is the density of ice, 922 kg m$^{-3}$. This effect is also very small as temperatures in the firn zone are increased by less than 0.001 $^{\circ}C$. Because neither of these mechanisms generates significant quantities of heat, they are ignored.

The thermal diffusivity, $k(\rho, \theta)$, is defined as:

$$k(\rho, \theta) = K(\rho, \theta) / c(\rho, \theta) \quad \text{m}^2 \ a^{-1}$$

(9)

where $K$ is the thermal conductivity. We assume the temperature and density dependence of the diffusivity are factorable and write:

$$k(\rho, \theta) = k_i (1 - 0.012 \theta) f(\rho)$$

(10)
where \( k_1 \) is the diffusivity of solid ice, \( 36.5 \text{ m}^2 \text{ a}^{-1} \), and the temperature coefficient is taken from Weller and Schwerdtfeger [1977]. The density dependence is more difficult to assess, as only a few direct measurements have been made (see Mellor, [1964] and Yen, [1981]). To avoid the problem of having to specify this dependence, we choose the upper ice sheet boundary in our calculations to be such that the diffusivity can be taken to be density independent. By numerically experimenting with different functional forms for \( f(\rho) \), we find that below 80m all the diffusivity models give nearly identical results. Thus we take the upper boundary in our ice sheet model to be at 80 m and \( f(\rho) = 1 \). The surface-temperature history that we specify is then the temperature at 80 m depth. Using different forms for \( f(\rho) \) between 0 and 80 m depth, we have found that the temperature change at the true surface is attenuated by only about 5% at 80 m. Thus the temperature warmings we calculate here are essentially identical to the true surface warmings.

Numerically, we solve the heat equation by making the Crank-Nicholson approximation and using an efficient matrix inversion scheme developed by R. Ewing (personal communication). We use a depth step of 10m, which gives identical results as a 5 m step, and a time step of 50 years. We have tested the Crank-Nicholson scheme used here against a simple forward-differencing solution. The two calculations gave identical results, but the forward differencing solution requires over 100 times as many iterations.

**ICE DYNAMICS**

One of the most important parameters in determining the temperature distribution in an ice sheet is the ice velocity (see Robin, [1955] for example). If we assume the flow near the Dome C ice divide is two dimensional, then the velocity components are related to the longitudinal strain rate, \( \dot{\varepsilon}(x,z) \), by:

\[
\begin{align*}
    v_x(x,z) &= \int_0^x \dot{\varepsilon}(x',z) dx' \\
    v_z(x,z) &= a - \int_0^z \dot{\varepsilon}(x,z') dz'
\end{align*}
\]  

with the horizontal velocity vanishing at \( x = 0 \), where the ice divide (and the Dome C camp) are located. If there is no basal sliding, we see that \( \dot{\varepsilon}(x,z) \)
must vanish at the base of the ice sheet. (The converse is not true, since
strictly speaking, equation (11) gives only the difference between the surface
velocity and the velocity at depth). The vertical velocity (or, equivalently,
the longitudinal strain rate) must be known as a function of depth in order to
solve the heat equation.

If the stress distribution in the ice sheet were known, the longitudinal
strain rate could be calculated from a flow law. The problem can be greatly
simplified by assuming that stress gradients along the flow direction can be
ignored. This implies that the ice thickness and surface slope are independent
of \( x \). If, in addition, one assumes that the longitudinal stress can be ignored
so the ice deforms in simple shear, then the flow is laminar (Paterson,[1981]
p. 87) and the horizontal velocity can be written:

\[
v_x(z) = \frac{2A(\rho ga)^{nH^{n+1}}}{(n+1)[1-(z/H)^{n+1}]} = v_{xs}[1-(z/H)^{n+1}]
\]

where \( \alpha \) is the surface slope, \( A \) is Glen's flow-law constant, and \( n \) is the flow-

law exponent. With these assumptions, the longitudinal strain rate is zero. To

find the lowest order correction, let \( \alpha \) and \( H \) be slowly varying functions of

\( x \). Then:

\[
\dot{\epsilon}(x,z) = \frac{\partial v_x}{\partial x} = v_{xs}[n(\alpha'/\alpha)(1-(z/H)^{n+1}) + (n+1)\alpha/H]
\]

Except perhaps very near the ice-sheet base, the term in \( \alpha/H \) for a reasonable

ice-sheet profile is negligible, so that the longitudinal strain rate can be

written:

\[
\dot{\epsilon}(x,z) = \dot{\epsilon}_s(x)[1 - (z/H)^{n+1}]
\]

As a result, \( \dot{\epsilon} \) and \( v_x \) vary with depth in the same way. We would expect equa-
tion (14) to be valid where the deformation is dominated by shearing. This as-
sumption breaks down at the ice divide where the surface slope vanishes. How-
ever, depending on the longitudinal stress distribution there, it is still pos-
sible that a form similar to equation (14) may describe the strain rate pro-

file at the divide.

The dynamics at and around an ice divide for an isothermal ice sheet fro-
zen to its bed have been studied using a finite-element model by Raymond
[1984]. He assumed the horizontal velocity profile was laminar (equation (12)) at a distance of 19 ice thickness from the divide and that the longitudinal strain rate there was given by equation (14). He found the ice flow remained laminar as the divide was approached, except for a transition zone within about 4 ice thickness of the divide. At the ice divide however, the longitudinal strain rate varied linearly with depth. Raymond also found that the width of the transition zone and the vertical velocity at the divide were relatively insensitive to the ice-sheet shape. However, if the ice rheology were linear viscous, the transition zone would vanish and there would be no difference between near-divide and far-divide flow. Unfortunately for our purposes here, the results above were no longer valid for a non-isothermal ice sheet.

Rather than attempt to solve for the stress distribution at the ice divide and then calculate the longitudinal strain rate from a flow law, here we take an approach similar to that in Thompson et al. [1982]: we assume the simplest dynamical behavior consistent with the boundary conditions. We assume that currently there is no bottom sliding, so that the longitudinal strain rate must vanish at the ice sheet base. We then consider a number of simple, monotonic functions for the longitudinal strain rate that satisfy this boundary condition. These functions are meant to model the actual depth dependence of the longitudinal strain rate, which is intimately related to factors such as temperature, stress, ice crystal orientation, and impurity content, as well as the specific form of the flow law. It should be noted that we make no assumptions concerning the actual stress distribution or the form of the flow law. We also ignore the effects of secular temperature variations in the ice sheet on the ice dynamics. In a later section we will examine the effect on calculated profiles of a time-dependent velocity profile resulting from changes in accumulation rate.

We consider two classes of functions such that the vertical strain rate vanishes at the bed (z=H in our coordinate system). Because of their simple behavior these functions can be characterized by a single parameter, the exponent p. We take the class A (laminar-like flow) strain-rate functions to be:

\[
\dot{\varepsilon}(z) = \dot{\varepsilon}_a (1 - \zeta^p)
\]

\[
\zeta = z/H
\]  

(15)

where \( \dot{\varepsilon}_a \) is the longitudinal strain rate at the ice sheet surface and \( \zeta \) is the
reduced depth. Some examples are shown in figure 3. For this class of functions the strain rate varies most rapidly with depth in the lower part of the ice sheet and the vertical gradient vanishes at the surface, except for \( p=1 \). From the discussion above, these class A functions are equivalent to laminar flow if \( p=n+1 \), where \( n \) is the exponent in Glen's flow law. From Raymond's results, these strain rate profiles might result at Dome C if the near-divide and far-divide flow were similar. This may be due to an effective linear viscous rheology or may be a result of divide migration due to a shift in the accumulation rate pattern [Weertman, 1973], so that the flow at depth at the current ice divide reflects the flow behavior of a region previously outside Raymond's transition zone.

It is interesting to note from figure 3 that for large values of \( p \), the strain rate is constant throughout much of the ice sheet, then decreases almost linearly to zero at the bed. This is similar to the Dansgaard-Johnsen model [Dansgaard and Johnsen, 1969], except that here the critical depth (i.e., the depth to which the strain rate remains constant) is a function of the exponent \( p \). For increasing \( p \), the critical depth increases, and as \( p \to \infty \) the strain rate becomes constant through the ice thickness and the vertical velocity becomes linear with depth.

Laminar-like flow functions may be more appropriate for the far-divide flow regime and it is possible that none of these functions will generate acceptable temperature profiles. As a result, we consider another general form for the longitudinal strain rate (many other forms are certainly possible and would be interesting to test). This is the class of simple polynomials for which the strain rate gradient with depth vanishes at the ice sheet base, rather than the surface as for laminar-like flow functions. For the class B (thin-skin flow) functions we take:

\[
\dot{\varepsilon}(z) = \dot{\varepsilon}_b (1 - \zeta)^p
\]

(16)

where again \( \dot{\varepsilon}_b \) is the surface strain rate and \( \zeta \) is the reduced depth. For large values of \( p \) these polynomials decrease rapidly in the upper ice sheet, yielding nearly stagnant ice at depth (see figure 4). As \( p \) increases, the deforming ice is confined to an increasingly thin surface layer, with \( p \to \infty \) characterizing a stagnant ice sheet. Paterson [1976], who made one of the few measurements of longitudinal strain rate as a function of depth in an ice sheet,
Figure 3.

The longitudinal strain rate as a function of depth for class A (laminar-like) flow for various values of the exponent p.
Figure 4.
The longitudinal strain rate as a function of depth for class B (thick-skin) flow for various values of the exponent p.
may have seen this thin-skin behavior three ice thicknesses from the divide on
the Devon Island Ice Cap. Within measurement error, the vertical gradient of
the longitudinal strain rate seems to go to zero at the ice sheet base; how-
ever, the measured profile is not a simple montonic function of depth, even
when firn compaction is accounted for.

These two classes of functions are related in the appropriate limits. We
see from equations (15) and (16) that p=OB (i.e., p=0, class B flow) is the
limiting function as p→A. Also, p=0A and p→B both characterize a stagnant
ice sheet. Both sets of functions also have another common member: p=1, the
strain rate function deduced by Raymond for an isothermal ice divide.

In the upper part of the ice sheet, vertical motion results from the in-
ternal deformation of fully densified ice and from firn densification. At Dome
C the densification effect is the dominant contribution to the vertical ve-
cocity above 50 m depth. At 80 m depth (the upper ice sheet boundary in our nu-
merical model), the contribution from densification is still significant. In
the appendix we derive an expression for the vertical velocity which includes
densification. The result is:

$$v_z(z) = \left( \rho_1 / \rho(z) \right) [\dot{\alpha} - (1/\rho_1) \int_{0}^{z} \dot{\epsilon}(z') \rho(z') dz']$$ (17)

where $\dot{\epsilon}(z)$ is the longitudinal strain rate function in fully densified ice,
and $\rho_1$ is the density of ice. Some examples for class A and class B flow are
shown in figure 5.

BASAL BOUNDARY CONDITION-

Based on Whillans' calculation of the response time of the East Antarctic
ice sheet, it seems that central East Antarctica at least has just begun to re-
spond to the Holocene warming. If this is the case, then the effects of the
warming are significant only in the upper part of the ice sheet, and it is rea-
sonable to assume that basal conditions have been unaffected by the climatic
transition. Thus we take the basal boundary condition to be time-independent.
To solve the heat equation we must specify either the basal temperature or tem-
perature gradient. Also, the vertical velocity at the base, which is equal to
the basal melt rate, must be specified.
Figure 5.

Vertical velocity as a function of depth for various values of the exponent $p$, calculated from equation (17) for class A flow (solid lines) and class B flow (dashed lines).
On the basis of airborne radio-echo soundings, Oswald [1975] found evidence for subglacial lakes over an extensive area around the Dome C ice divide. He suggested that melting is not confined to the specific lake sites, but is widespread so that much of the basal ice in this area is at the pressure melting point.

Surface radar studies by Jezek and Shabtie (personal communication) showed weak radar reflections around the Dome C camp, which could be interpreted as the absence of basal water. However, at 25 to 30 km from camp, reflection strengths were highly variable over distances on the order of 1 km, suggesting the presence of small pockets of water. Based on these results, it is likely that the basal ice at Dome C is at, or at least near, the pressure melting point. The absence of subglacial water may be due to the absence of basal melting, or perhaps the melt rate is small and the base is well-drained.

Here we assume that the base is at the pressure-melting point, -2.6 °C, and that if any basal melting is occurring, the melt rate is small enough such that the vertical velocity can be taken to vanish at the base. While this view is consistent with the airborne and surface-radar data, other interpretations are possible. For example, basal melt rates could be high under the ice near the borehole, with the meltwater being drained in narrow channels.

The calculated temperature profiles give the heat flux at the ice sheet base, and the basal melt rate could then be calculated if the geothermal heat flux were known (see equation 21). (To be consistent with our assumed basal boundary condition, we would have to disregard climatic transition models which gave rise to temperature profiles with "large" basal melt rates.) Unfortunately, the geothermal flux in this area is poorly known. A global heat-flow map constructed by Chapman and Pollack [1975] based on tectonic setting and age predicts that the geothermal flux over most of the East Antarctic Archean-Proterozoic shield is on the order of 30-40 mw m⁻². This map also shows the Dome C area located near a province characterized by a geothermal flux of about 40-50 mw m⁻², while in the coastal Wilkes land area, predicted values range between 50-60 mw m⁻².

By modifying the analytic temperature solution developed by Robin [1955] to include basal melting, we can estimate the geothermal flux as a function of basal melt rate. In the Robin model, the longitudinal strain rate is constant with depth (our class B, p=0 function) and with basal melting the vertical velocity becomes:

\[ v_z(z) = -\left[ \hat{\alpha}_m + (\hat{\alpha}-\hat{\alpha}_m)z/H \right] \]  

(18)
where $\dot{a}_m$ is the basal melt rate in meters of ice per year, and where the $z$-coordinate is now positive upward from the base. Ignoring horizontal advection and taking the thermal diffusivity as constant reduces the heat equation to a one-dimensional form that can be integrated easily with the result:

$$\theta(z) = (\partial\theta/\partial z)_b \left( \frac{3}{2}2b \right) \exp(b^2\zeta^2) \{\text{erf}[b(z+\xi)] - \text{erf}(b\xi)\} + \theta_b \tag{19}\$$

where the subscript $b$ refers to basal values, and where

$$b^2 = \left(\frac{(\dot{a}-\dot{a}_m)}{2Hk}\right) \tag{20}\$$

$$\zeta = \frac{\dot{a}_m H}{(\dot{a}-\dot{a}_m)} \tag{20}\$$

Our numerical studies suggest that $k$, the average thermal diffusivity through the ice sheet, is about $54 \text{ m}^2 \text{ a}^{-1}$. With the current mean surface temperature at $-53.7 \text{ °C}$, and the base at the pressure melting point, equation (19) fixes the basal gradient at $-0.0214 \text{ °C m}^{-1}$ and the heat flux into the ice sheet becomes about $48 \text{ mW m}^{-2}$. If there is no basal melting, the basal flux equals the geothermal flux, which is then in reasonable agreement with the value predicted by Chapman and Pollack. If basal melting is occurring, then the basal temperature gradient, $(\partial\theta/\partial z)_b$, the geothermal flux $G$, and the basal melt rate are related by:

$$G - Lp_1 \dot{a}_m = K(\partial\theta/\partial z)_b \tag{21}\$$

where $L$ is the latent heat of fusion, $3.2 \times 10^5 \text{ J kg}^{-1}$. For the basal gradient calculated above, small amounts of basal melting (say on the order of $0.1-0.2 \text{ mm a}^{-1}$) require a geothermal flux around $50 \text{ mW m}^{-2}$, while for basal melt rates on the order of $1-2 \text{ mm a}^{-1}$, equation (21) gives a geothermal flux of $65-70 \text{ mW m}^{-2}$, significantly larger than predicted for the supposed tectonic setting. Rao and Jessop [1975] point out, however, that the geothermal flux over a given shield can vary by a factor of 2-3, and that the lateral scale of heat flow anomalies, while typically on the order of 50 to 100 km, can be as small as 25 km. Thus it is not impossible that basal melt rates could be significantly larger than we suppose. The calculation above shows however that our simple assumptions concerning the basal conditions are consistent with what is currently known about the East Antarctic basement in the Dome C area.
If East Antarctica were a typical pre-Cambrian shield with a geothermal flux of around 40 mW m\(^{-2}\), then equation (19) would fix the basal temperature at about -11 °C. Because the accumulation rate, surface temperature, and ice thickness change very little from the camp site to 25 km away where bright radar reflections are seen, it is unlikely that the basal ice at Dome C could be so cold if melting is occurring only 25 km away.

**INITIAL CONDITION**

To solve the heat equation, the initial temperature distribution prior to the onset of the glacial-interglacial transition must be specified. This pre-transition temperature distribution reflects the integrated response of the ice sheet to earlier climatic events, and need not have been a steady-state distribution. In this section however, we will demonstrate that this temperature distribution probably was nearly stationary.

The past Wisconsinan Glacial stage seems to have included a cold period around 60,000 y.b.p., followed by a period around 40,000 y.b.p. that may have been as warm as the current interglacial [Budd, 1979]. If the response time of the East Antarctic ice sheet is as long as 35,000 years, then it may be that the current temperature profile in the upper 800 m may be strongly influenced by climatic events during the early and mid-Wisconsinan glacial stage.

Using our numerical model, we have calculated the rate of temperature change with time resulting from two hypothetical 5 °C warmings, one beginning at 45,000 y.b.p. and the other at 15,000 y.b.p. The warming in both cases was linear with time over 5,000 years and the strain rate was taken to vary linearly with depth. The accumulation rate and ice sheet thickness were constant at their current values, and the basal temperature was fixed at -2.6 °C. The results are shown in figure 6.

In the lower 800 m, the variation of temperature with time was comparable for both warmings, being strongly constrained by the fixed basal temperature. In the upper 1000 m dθ/dt from the more recent warming was nearly an order of magnitude larger. Thus the signal from a major climatic event during the mid-Wisconsinan may still be present, but this event would have occurred too far in the past to have had a significant effect on the current profile. Of course, the actual surface temperature history is no doubt more complicated than that used in this simple numerical experiment and changes in the accumula-
Figure 6.

Time rate of change of temperature as a function of depth resulting from a warming of $5\,^\circ C$ over 5,000 years starting 15,000 years ago (line A) and $5\,^\circ C$ over 5,000 years starting 45,000 years ago (line B). Ice sheet thickness, accumulation rate, and basal temperature were the same in both cases, and the strain rate was linear with depth.
tion rate and ice thickness, which have been ignored, also can alter the distribution of temperature with time. Very little information is available as to what these changes might be. Also there is no indication from the Dome C oxygen-isotope profile for a climatic event approaching the magnitude of that at the Wisconsinan-Holocene transition over the 30,000 to 40,000 year period spanned by the Dome C core.

Based on these considerations, we assume that any non-stationary character in the current temperature profile was generated at the past glacial-interglacial transition, and as a result, the temperature distribution just prior to the Holocene warming was a stationary distribution. To calculate the pre-warming temperature profile, we first specify a linear temperature profile with the upper boundary at the appropriate late glacial temperature. With a time step of 1,000 years, we find that 160 steps is sufficient to generate a profile which changes by less than 0.0002 °C between time steps.

SURFACE BOUNDARY CONDITION-

In order to determine the velocity distribution on the basis of agreement between calculated and measured temperature profiles, it is necessary to specify the surface boundary condition as completely as possible. This means specifying the surface temperature as a function of time. Complications arise, however, if the accumulation rate is also time-dependent because the stress distribution in the ice sheet may be altered as a function of time, resulting in secular changes in velocity and ice thickness. Also the age of ice at depth is related to the accumulation rate through the vertical velocity profile, and as a result, variations in the accumulation rate have implications for the ages of climatic events inferred from the ice-stratigraphy.

Surface Temperature Record

The stable oxygen-isotope profile (see figure 7) measured on the 900 m Dome C core [Lorius et al., 1979] is believed to contain a proxy surface temperature record at the borehole site. While the early work of Dansgaard [1964] demonstrated a relation between the oxygen isotopic composition of precipitation and surface temperature on a global scale, the exact form of this relation in the Antarctic remains unclear. This is because processes unrelated to
Figure 7.

The Dome C oxygen isotope profile. Raw data were taken from Ritz [1982] and smoothed using a low-pass filter. The arrows point to features that were correlated with oxygen isotope features on the Indian Ocean marine sediment core MD 73025.
mean annual surface temperature may strongly affect the isotopic signal recorded in the ice sheet. In regions of low accumulation rate, deposition noise caused by the redistribution and alteration of surface snow by sun and wind may be the dominant component in the mean annual isotopic record [Robin, 1983]. If the isotopic signal is averaged over an appropriately large period, then deposition noise and other stochastic influences should be filtered out. At Dome C the isotopic records from adjacent cores correlate poorly even when both records have been smoothed with a 500 year filter [Benoist et al., 1982]. This suggests that the appropriate averaging period needed to obtain the temperature component in the isotopic record may be even longer.

As a result, we assume here that only the gross shape of the isotopic profile reflects the Dome C surface-temperature history. Following Lorius et al., we take the Wisconsinan to Holocene transition to begin in ice now at 510 m depth and to end at 380 m depth. We approximate the surface-temperature record by taking the mean surface temperature to be constant prior to the onset of the Holocene warming and to be constant at the current mean surface temperature for the postwarming period. During the glacial-interglacial transition, we take the mean surface temperature to vary linearly with time (see figure 8). We also let the magnitude of the surface warming be a free parameter in this study. This will enable us to determine the relation between the surface warming and the ice dynamics we assume here (if one exists), so as suggest the velocity profile most consistent with a particular interpretation of the oxygen isotope record.

The age of the ice at 510 m and 380 m depth, corresponding to the beginning and end of the glacial-interglacial transition, is ultimately determined by the ice flow. If the velocity distribution in the ice sheet is stationary with time, at an ice divide the age of the ice at any depth can be calculated from the vertical velocity distribution according to:

$$T(z) = \int_0^z \frac{dz'}{v_z(z')}$$  \hspace{1cm} (22)

In our study, the velocity distribution assumes additional significance in that it not only affects the distribution of temperature in the ice sheet through the advective transfer of heat, but is a factor in specifying the surface temperature boundary condition.
Figure 8.

The time dependence of the surface temperature at Dome C (solid line), based on the oxygen isotope profile (dashed line). The depth-age relation used here is for $p=1$, $\Delta = 0.040 \text{ m a}^{-1}$. 
Accumulation Rate Record

By correlating selected features on the Dome C oxygen isotope profile with similar features on a dated marine sediment core (core MD73025) taken from the East Indian Ocean [Duplessy, 1978], Lorius et al. attempted to deduce accumulation rate variations at Dome C. This cross-correlation resulted in three independent dates on the Dome C core: 15,500±500 yrs at 510 m depth, 12,600±300 yrs at 437 m depth and 10,500±300 yrs at 380 m depth. They assumed that the vertical velocity was linear with depth and proportional to the accumulation rate, and then used equation (22) to calculate the accumulation rates needed over the three depth intervals to make the ice-core and marine-core time scales coincide. This led to a 30% increase in accumulation rate, from 0.029 m a⁻¹ to 0.037 m a⁻¹ over the past 15,500 years. The time scale that results gives the age of the ice at the bottom of the core to be about 30,000 years.

Other evidence on the accumulation rate record at Dome C comes from the work of Thompson et al. [1979]. They determined the annual layer thickness in 51 sections of the 900 m Dome C core by identifying and measuring the separation between annual microparticle peaks. In figure 9, we show their smoothed results for the average annual layer thickness as a function of depth. The overall trend in the upper 900 meters is that the layer thickness is nearly constant with depth, and there seems to be little apparent thinning as layers are buried. On the same figure, we also show the annual layer thickness as a function of depth for two constant strain rate ice flow models. For one, the accumulation rate is taken constant at the current value, while for the other we use the accumulation rates calculated by Lorius et al. and a simple non-steady state model (to be discussed in a later section). We note that the measured layer thicknesses seem to be more consistent with the time-independent accumulation rate model. However, this is also a result of the constant strain rate used here; other class A and class B strain rate functions result in more pronounced layer thinning with depth.

As a result, we consider three alternatives for the accumulation rate change with time since the late Wisconsinan. In the first approach (fixed-current-value model), we simply take the accumulation rate to be constant with time at its current value. The ice sheet thickness and velocity distribution are then also time independent. In the second approach (fixed-average-value
Figure 9.

Average annual layer thickness as a function of depth measured in 51 sections of the Dome C core by Thompson et al. [1979]. Error bars show the range in maximum and minimum values at various depths. Dashed lines show calculated values for the strain rate constant and \( \dot{\alpha} = 0.040 \, \text{m a}^{-1} \) (line 1) and \( \dot{\alpha} \) varying according to the Lorius et al. [1979] reconstruction (line 2).
model), we take the accumulation rate to be constant, but adjust its value for each strain rate function so that the age at 510 m depth is 15,500 year. The third alternative (step-change model) is to use the marine core sediment dates and derive accumulation rate changes for each strain rate function in the same way as LORIUS et al. Once again each strain rate function will yield a different set of accumulation rate values for the three time intervals. In all cases however, the surface temperature depends linearly on time during the climatic transition period and is time independent before and after.

Fixed-current-value model—

In this model we take the accumulation rate to be constant at its current value: 0.040 m a\(^{-1}\). As a result, the ice sheet thickness and the velocity profile for each strain rate function is independent of time. Since the 510 m and 380 m levels on the oxygen isotope profile are ice-equivalent depths, times for the start and end of the climatic transition are calculated from equations (17) and (22) with \(\rho(z)=\rho_1\). Some derived times for the onset and end of the glacial-interglacial transition are shown in table 1.

Fixed-average-value model—

As mentioned earlier, the correlation of selected features on the oxygen isotope profiles from Dome C and the marine sediment core MD 73025 gives three dates for the Dome C depth-age profile. In this model we again take the accumulation rate to be independent of time, but adjust the value of \(\dot{\alpha}\) so that for each velocity profile, the age at 510 m depth is 15,500 years. From equation (17) with \(\rho(z)=\rho_1\):

\[
v_z(z) = \dot{\alpha} - \int_0^z \dot{c}(z')dz'
\]

(23)

We take the vertical velocity to vanish at the base and from equations (23), (15), and (16) we have for class-A and class-B flow:

\[
\begin{align*}
v_z(z)_a &= \dot{\alpha}[1 - \xi - (\xi/p)(1 - \xi^p)] = \dot{\alpha} F_a(\xi, p) \\
v_z(z)_b &= \dot{\alpha}(1 - \xi)^{p+1} = \dot{\alpha} F_b(\xi, p)
\end{align*}
\]

(24)
Table 1. Timing of Holocene warming for fixed current accumulation rate

Age of ice at 510 and 380 m depth, corresponding to the onset and end of the Holocene warming, is shown for class A (laminar-like) and class B (thin-skin) flow as a function of the exponent p. The accumulation rate is fixed at the current value, 0.040 m a^{-1}. Values are rounded to the nearest 50 years.

Class A \[ \dot{\varepsilon}_z = (p+1)/p \left(\dot{a}/H\right)(1 - (z/H)^p). \]

<table>
<thead>
<tr>
<th>p</th>
<th>T(510),y.b.p.</th>
<th>T(380),y.b.p.</th>
<th>trans. period, a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>16,400</td>
<td>11,500</td>
<td>4,900</td>
</tr>
<tr>
<td>0.50</td>
<td>15,750</td>
<td>11,100</td>
<td>4,650</td>
</tr>
<tr>
<td>0.75</td>
<td>15,300</td>
<td>10,850</td>
<td>4,450</td>
</tr>
<tr>
<td>1.00</td>
<td>15,000</td>
<td>10,700</td>
<td>4,300</td>
</tr>
<tr>
<td>2.00</td>
<td>14,450</td>
<td>10,400</td>
<td>4,050</td>
</tr>
<tr>
<td>3.00</td>
<td>14,200</td>
<td>10,300</td>
<td>3,900</td>
</tr>
<tr>
<td>4.00</td>
<td>14,100</td>
<td>10,250</td>
<td>3,850</td>
</tr>
<tr>
<td>5.00</td>
<td>14,050</td>
<td>10,200</td>
<td>3,850</td>
</tr>
</tbody>
</table>

Class B \[ \dot{\varepsilon}_z = (p+1) \left(\dot{a}/H\right)(1 - z/H)^p. \]

<table>
<thead>
<tr>
<th>p</th>
<th>T(510),y.b.p.</th>
<th>T(380),y.b.p.</th>
<th>trans. period, a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>13,800</td>
<td>10,000</td>
<td>3,800</td>
</tr>
<tr>
<td>0.50</td>
<td>14,400</td>
<td>10,400</td>
<td>4,000</td>
</tr>
<tr>
<td>1.00</td>
<td>15,000</td>
<td>10,700</td>
<td>4,300</td>
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<tr>
<td>1.50</td>
<td>15,650</td>
<td>11,000</td>
<td>4,650</td>
</tr>
<tr>
<td>2.00</td>
<td>16,300</td>
<td>11,300</td>
<td>5,000</td>
</tr>
<tr>
<td>2.50</td>
<td>17,000</td>
<td>11,700</td>
<td>5,300</td>
</tr>
<tr>
<td>3.00</td>
<td>17,800</td>
<td>12,100</td>
<td>5,700</td>
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<td>4.00</td>
<td>19,500</td>
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<td>6,500</td>
</tr>
<tr>
<td>4.50</td>
<td>20,400</td>
<td>13,400</td>
<td>7,000</td>
</tr>
</tbody>
</table>
and the vertical velocity is directly proportional to the accumulation rate. The adjusted accumulation rate, $\hat{a}_{\text{ave}}$, is calculated from equation (22):

$$
\hat{a}_{\text{ave}} = \frac{1}{T(510)} \int_{0}^{510} \frac{dz'}{F_{a,b}(\zeta', p)}
$$

where $T(510) = 15,500$ years. Derived values for $\hat{a}_{\text{ave}}$ (the average accumulation rate over the past 15,500 years) are shown in table 2, along with the derived time for the end of the transition and the transition time. We see that for all class-A functions and for class-B functions with $p \leq 2.5$, derived average accumulations rates agree with the current measured value within the quoted error. Note also that for $p = 2.0B, 2.5B$, and $0.25A$, derived transition times agree with the 5,000 year interval implied by the oxygen isotope profile cross-correlation.

Step-change model-

In this model we fix the beginning and end of the climatic transition at 15,500 y.b.p. and 10,500 y.b.p., as suggested by the marine-sediment dates, and let the accumulation rate vary with time. We can define average accumulation rates over the time intervals $\Delta t_1 = 0-10,500$ years, $\Delta t_2 = 10,500-12,600$ years, and $\Delta t_3 = 12,600-15,500$ years, so that the time variation in accumulation rate occurs in two step changes at 12,600 and 10,500 years. From equation (25):

$$
\hat{a}_1 = \frac{1}{\Delta t_1} \int_{z_j}^{z_k} \frac{dz'}{F_{a,b}(\zeta', p)}
$$

where $\hat{a}_1$ is the accumulation rate in the time interval $\Delta t_1$ and $z_j$ and $z_k$ are the ice equivalent depths at the beginning and end of each $\Delta t_1$. Derived values of $\hat{a}_1$ are given in table 3. The initial steady-state temperature profile for each strain rate function is calculated using the $\hat{a}_1$ calculated over $\Delta t_3$, which we also take to be the accumulation rate during the late Wisconsinan glacial stage. From table 3 we see that all class-A functions imply an accumulation rate increase since the end of the Wisconsinan glacial, while for $p > 3B$, derived accumulation rates were greater during the late Wisconsinan than now.
Table 2. Fixed average value accumulation rates.

The average accumulation rate needed to give an age of 15,500 a at 510 m depth is shown as a function of the exponent p for class A and class B flow. Also shown is the age at 380 m depth, corresponding to the end of the warming, and the transition period.

Class A $\dot{e}_z = (p+1)/p (\dot{a}/H)(1 - (z/H)^p)$.

<table>
<thead>
<tr>
<th>p</th>
<th>$\dot{a}_{ave}, m a^{-1}$</th>
<th>T(380), y.b.p.</th>
<th>trans. period, a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.0424</td>
<td>10,850</td>
<td>4,650</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0405</td>
<td>10,950</td>
<td>4,550</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0395</td>
<td>11,000</td>
<td>4,500</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0387</td>
<td>11,050</td>
<td>4,450</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0372</td>
<td>11,150</td>
<td>4,350</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0367</td>
<td>11,200</td>
<td>4,300</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0364</td>
<td>11,250</td>
<td>4,250</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0363</td>
<td>11,250</td>
<td>4,250</td>
</tr>
</tbody>
</table>

Class B $\dot{e}_z = (p+1)(\dot{a}/H)(1 - z/H)^p$.

<table>
<thead>
<tr>
<th>p</th>
<th>$\dot{a}_{ave}, m a^{-1}$</th>
<th>T(380), y.b.p.</th>
<th>trans. period, a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0356</td>
<td>11,300</td>
<td>4,200</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0371</td>
<td>11,200</td>
<td>4,300</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0387</td>
<td>11,050</td>
<td>4,450</td>
</tr>
<tr>
<td>1.50</td>
<td>0.0404</td>
<td>10,900</td>
<td>4,600</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0421</td>
<td>10,800</td>
<td>4,700</td>
</tr>
<tr>
<td>2.50</td>
<td>0.0440</td>
<td>10,700</td>
<td>4,800</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0459</td>
<td>10,500</td>
<td>5,000</td>
</tr>
<tr>
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<td>0.0480</td>
<td>10,400</td>
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<tr>
<td>4.00</td>
<td>0.0502</td>
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<td>5,200</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0550</td>
<td>10,000</td>
<td>5,500</td>
</tr>
</tbody>
</table>
Table 3. Step-change accumulation rates

Derived accumulation rates in the step-change model, based on the correlation between the Dome C and marine sediment core MD73025 oxygen isotope profiles. The Holocene warming begins at 15,500 y.b.p. and ends at 10,500 y.b.p. The calculated values $\dot{a}_1$, $\dot{a}_2$, and $\dot{a}_3$ are the average accumulation rates in the time intervals 0-10,500 a, 10,500-12,600 a, and 12,600-15,500 a respectively. All resulting depth-age relations have three points in common: 10,500 years at 380 m, 12,600 years at 437 m and 15,500 years at 510 m.

Class A $\dot{z} = \frac{(p+1)}{p} \frac{(\dot{a}/H)(1 - (z/H)^p)}{1 - (z/H)^p}$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\dot{a}_1$, (m a$^{-1}$)</th>
<th>$\dot{a}_2$, (m a$^{-1}$)</th>
<th>$\dot{a}_3$, (m a$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0401</td>
<td>0.0391</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0421</td>
<td>0.0378</td>
<td>0.0367</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0413</td>
<td>0.0363</td>
<td>0.0351</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0407</td>
<td>0.0353</td>
<td>0.0340</td>
</tr>
<tr>
<td>2.00</td>
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</tr>
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<td>3.00</td>
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<td>0.0325</td>
<td>0.0309</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0389</td>
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<td>0.0305</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0388</td>
<td>0.0319</td>
<td>0.0303</td>
</tr>
</tbody>
</table>

Class B $\dot{z} = \frac{(p+1)}{p} \frac{(\dot{a}/H)(1 - (z/H)^p)}{1 - (z/H)^p}$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\dot{a}_1$, (m a$^{-1}$)</th>
<th>$\dot{a}_2$, (m a$^{-1}$)</th>
<th>$\dot{a}_3$, (m a$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0383</td>
<td>0.0316</td>
<td>0.0293</td>
</tr>
<tr>
<td>0.50</td>
<td>0.0394</td>
<td>0.0337</td>
<td>0.0315</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0407</td>
<td>0.0353</td>
<td>0.0340</td>
</tr>
<tr>
<td>1.50</td>
<td>0.0418</td>
<td>0.0383</td>
<td>0.0366</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0432</td>
<td>0.0401</td>
<td>0.0395</td>
</tr>
<tr>
<td>2.50</td>
<td>0.0445</td>
<td>0.0435</td>
<td>0.0426</td>
</tr>
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<td>3.00</td>
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<td>0.0495</td>
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<td>0.0519</td>
<td>0.0533</td>
</tr>
<tr>
<td>4.50</td>
<td>0.0505</td>
<td>0.0562</td>
<td>0.0575</td>
</tr>
</tbody>
</table>
For this climate-transition model the ice sheet thickness and velocity profile are no longer strictly time-independent. The effect of the derived accumulation rate changes on the ice thickness could be estimated using an ice sheet response model. But the effect of a time-varying accumulation rate on the velocity distribution is more difficult to assess. Here we assume that ice sheet thickness changes can be ignored. In the next section we give a simple non-steady model to calculate the lowest order correction to the steady state velocity distribution. This approach is motivated by two considerations. One is that our numerical model becomes much more complex if the upper boundary is no longer fixed in space. The other is that our modeling shows that the temperature distribution is sensitive to small changes in the accumulation rate (see also Ritz, [1982]) that would have little effect on the ice sheet thickness (at least over the time period of interest here). It should be kept in mind that our non-steady state results then are strictly applicable to situations where the ice sheet thickness change is small.

NON-STeadY-STATE MODEL-

A change in accumulation rate or mass balance leads to a change in ice sheet thickness as the mass flux is altered. As a result, the distribution of stress is affected and the ice flow speeds up or slows down with time until stability is restored. This process was first considered in detail by Nye [1960,1963,1963a]. Using a perturbative approach, Nye showed that the ice sheet response to changes in mass balance involved the generation of kinematic waves which propagated down the glacier and ultimately restored stability. Whillans [1981] used a similar approach, but included the effects of temperature on the shear strain rate. Whillans also used borehole tilting results from Camp Century and Byrd Station to specify the depth dependence of the shear strain rate, while Nye used Glen's flow law. Except for the case where the strain rate may be independent of depth, neither approach addressed the question of how the distribution of strain rate or velocity with depth in the ice sheet changed with time in response to variations in accumulation rate.

Here we present a simple model for determining the change in the vertical distribution of the longitudinal strain rate and the vertical velocity, based on mass continuity. We again assume two-dimensional flow, so that mass continuity implies at any time t:
\[
\dot{\alpha}(x,t) = \sum_x \int_{0}^{H} v_{x}(x,z') dz' = \partial_z H(x,t)
\]

(27)

Taking the derivative of the integral gives:

\[
\partial_x \int_{0}^{H} v_{x}(x,z') dz' = v_{xs}(x) \partial_x H(x,t) + H(x,t) \partial_z \dot{\varepsilon}(x,t)
\]

(28)

where the average longitudinal strain rate is defined as:

\[
\dot{\varepsilon}(x,t) = \frac{1}{H(x,t)} \int_{0}^{H} \dot{\varepsilon}(x,z',t) dz'
\]

(29)

From equations (27), (28), and (29) we then can write:

\[
\tilde{\varepsilon}(x,t) = [\dot{\alpha}(x,t) - v_{xs}(x) \partial_x H(x,t) - \partial_z H(x,t)]/H(x,t)
\]

(30)

As we approach the ice divide, the surface slope, \(\partial_x H(x,t)\), vanishes so that equation (30) becomes:

\[
\tilde{\varepsilon}(t) = [\dot{\alpha}(t) - \partial_z H(t)]/H(t)
\]

(31)

Now we assume that the longitudinal strain rate can be written in a factorized form as

\[
\dot{\varepsilon}(z,t) = \tilde{\varepsilon}(t) \psi(z,H)
\]

(32)

where \(\psi(z,H)\) is normalized according to:

\[
(1/H) \int_{0}^{H} \psi(z',H) dz' = 1
\]

(33)

Equation (32) is likely to be true for small changes in accumulation rate where changes in ice thickness can be ignored. It may also be necessary that the accumulation rate vary slowly with time, so that the stress distribution at any time in the ice sheet is nearly steady-state. Assuming then we can ignore secular changes in ice thickness, from equations (31) and (32) we have finally:

\[
\dot{\varepsilon}(z,t) = [\dot{\alpha}(t)/H] \psi(z,H)
\]

(34)
Since we are assuming that the position of the ice sheet surface is fixed with time, the surface boundary condition on the vertical velocity remains that \(v_z(z=0) = \dot{a}(t)\). We can then use equation (17) to calculate the vertical velocity with the result:

\[
v_z(z,t) = \dot{a}(t)[1 - (1/H) \int_0^z \psi(z',H)dz']
\]

At this level of approximation then we see that the vertical velocity is simply proportional to the (time-dependent) accumulation rate. Obviously for large changes in accumulation rate the equation above is not reasonable, as we hardly expect an instantaneous doubling of accumulation rate to lead to a simultaneous doubling of vertical velocity at all depths in the ice sheet. But for small changes in accumulation rate that occur slowly with time, we expect equation (35) to provide the lowest order correction for the depth dependence of the ice sheet response.

We can also use equation (35) to calculate the depth-age relation. By definition of the vertical velocity:

\[
v_z(z,t) = \frac{dz}{dt}
\]

we then have:

\[
\int \dot{a}(t) dt = \int_0^z [1 - (1/H) \int_0^{z'} \psi(z',H)dz']^{-1}
\]

and the depth-age relation results from solving equation (37) for \(t\). For a constant accumulation rate, we recover the usual expression for the depth-age relation. In the case where the time dependence of the accumulation rate can be specified by a series of step changes, the above equation also becomes simple.

In particular, consider the problem where independent dating of the ice (here obtained by cross-correlation with the dated marine sediment core) gives the ages \(T_i\) at depths \(z_i\). For a given \(\psi(x,z)\) and assuming that the accumulation rate changes stepwise at times \(T_i\), we wish to derive values for the accumulation rate over the time intervals \(\Delta T_i = T_i - T_{i-1}\), where \(T_0=0\). Define the step function, \(s(x)\), with the property:

\[
s(x) = \begin{cases} 
1, & x \geq 0 \\
0, & x < 0 
\end{cases}
\]

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and let \( \dot{a}_i \) be the accumulation rate in the time interval \( \Delta T_i \). We can write then:

\[
\dot{a}(t) = \dot{a}_i [s(t-T_i-1) - s(t-T_i)]
\]

(39)

Then for all time intervals:

\[
\dot{a}(t) = \sum_{i=0}^{N} \dot{a}_i [s(t-T_i-1) - s(t-T_i)]
\]

(40)

and from equation (37) for \( t=0 \) to \( t=t_N \), we have:

\[
\sum_{i=0}^{N} \int_{T_{i-1}}^{T_i} [s(t-T_i-1) - s(t-T_i)] \, dt = \int_{z^N}^{z} \int_{0}^{z} \left[1-(1/H) \int_{z'}^{z} \psi(z',H) \, dz' \right]^{-1} \, dz
\]

(41)

If we consider the depth interval \( z_{i-1} \) to \( z_i \) and the corresponding time interval \( T_{i-1} \) to \( T_i \), the above equation becomes:

\[
\dot{a}_i = (H/\Delta T_i) \int_{z_{i-1}}^{z_i} [H - \int_{z'}^{z} \psi(z',H) \, dz']^{-1} \, dz
\]

(42)

which is the same as equation (26). As discussed above, this equation is likely to be valid only if accumulation rate changes are small enough so that changes in ice thickness can be ignored.

RESULTS AND DISCUSSION-

Best Fit Criterion and Derived Surface Warmings-

To determine the best fit profile for a given strain rate function and climatic transition model, we varied the surface temperature change until a calculated profile was obtained that satisfied a best fit criterion. Let

\[
\delta \theta_i = \theta(z_i) - \theta(z_i)
\]

(43)

where the subscripts denote the calculated and measured values. Suppose \( |\delta \theta_i|_{\text{max}} \) is the largest (absolute) value for a given profile. The derived surface warming is the value that causes \( |\delta \theta_i|_{\text{max}} \) to be minimized. Some examples
of $\delta \theta_1$ versus depth for best fit profiles are shown in figure 10. For all best fit profiles, the variation of $\delta \theta_1$ with depth has the same general shape. Calculated temperatures tend to be warmer than those measured in the upper and lower few hundred meters and colder in between. Of course the values of $\delta \theta_1$ depend partly on our choice of a best fit criterion. However, some numerical experimentation shows that if we assume surface temperature and $\hat{\alpha}$ to been time-independent since the onset of the Holocene, then the trends in $\delta \theta_1$ with depth are independent of the detailed time variation of temperature and accumulation rate during the climatic transition. The shape of the $\delta \theta_1$ curve may perhaps be related then to a more recent climatic change or sequence of changes, although the relevant time scale is not apparent.

The magnitude of the derived warming clearly depends on our choice of a best-fit criterion. Another criterion considered was selecting those profiles that minimized the standard deviation between measured and calculated values. Since there are only three measured values in the lower 200 meters, we chose to minimize the standard deviation in the upper 600 meters where the data are more equally spaced. While it was possible to generate profiles with a standard deviation less than 0.02 °C, this resulted in large positive $\delta \theta_1$ in the lower 200 meters. Because there was no compelling reason to fit one part of the measured profile at the expense of another, this criterion was discarded. However, the differences in derived surface warmings that resulted from the two criteria were small, typically around 0.3 to 0.4 °C.

The best-fit surface-temperature warmings as a function of the exponent $p$ are shown in figure 11 for class A flow and figure 12 for class B flow. The two flow models show quite different general trends: warmings increase as a function of $p$ for class B flow and decrease for class A flow. We note also that derived warmings from the non-steady state (NSS) step-change model and the steady-state current-value model are very similar for class A flow, and for class B flow, derived warmings from the average-value and step-change models are nearly equal. For class A flow, from tables 1 and 3 we see that in general $\hat{\alpha}_{\text{cur}}$ and $\hat{\alpha}_1$ are almost the same and the transition times for the step-change and current-value models are similar for each $p$. For class B flow transition times and accumulation rates ($\hat{\alpha}_1$ and $\hat{\alpha}_{\text{ave}}$) are very close for the step-change and average-value models. The transition time and accumulation rate over the past 10,500 years seem to be key parameters in determining the temperature distribution for a given flow exponent $p$. Calculated warmings are rela-
Figure 10.

Representative temperature profile fits with $\delta \theta_i$ being the difference between calculated and measured temperatures. For curve I, $p=3.5B$, $\dot{a}=0.048$ m a$^{-1}$, for curve II, $p=3.5B$, $\dot{a}=0.040$ m a$^{-1}$, and for curve III, $p=1A$, step-change model (see table 3 for values).
Figure 11.
Derived surface warmings as a function of the exponent $p$ for class A flow. Line A is the fixed current-value model ($\dot{a}=0.040 \text{ m a}^{-1}$), line B is the fixed average value model (see table 2) and line C is the step-change model (see table 3).
Figure 12.

Derived surface warmings as a function of $p$ for class B flow. Line A refers to the fixed current-value model, B refers to the fixed average-value model, and C refers to the step-change model.
tively sensitive to the post-warming accumulation rate because this fixes the
advective transfer of heat for over two-thirds of the time interval of
interest.

Ritz [1982] has deduced the velocity distribution at Dome C in the con-
text of a coupled temperature-velocity calculation. The current measured tem-
perature distribution is assumed to be stationary, and an iterative scheme
with velocity and temperature coupled via Glen's flow law is used. The result
is that except in the very bottom of the ice sheet, the strain rate is con-
stant and the vertical velocity can be approximated by:

\[ v_z(z) = \left( \frac{\dot{a}}{H} \right) (z - z_m) \]  (44)

where \( z_m \) is a parameter that depends mainly on the geothermal gradient and the
accumulation rate. Our results here are in good qualitative agreement, as we
also find that if the current profile is a steady-state or nearly steady-state
temperature profile (i.e., little or no surface temperature change since the
end of the Wisconsinan glacial), then the corresponding strain-rate functions
that generate these profiles are nearly constant with depth.

Derived Geothermal Heat Flux-

Because we assume that no basal melting is occurring, the basal heat flow
is the same as the geothermal flux. Figure 13 shows the basal flux derived
from the basal temperature gradient plotted as a function of the flow exponent
p. For the different ice-flow and climatic-transition models considered, the
basal flux ranges only from 53 to 58 mw m\(^{-2}\). The differences reflect the basal
temperature gradient in the steady-state, initial-condition ice sheet; this is
a function of the late-Wisconsinan upper-boundary temperature and the effects
of vertical advection for each strain rate function. It is interesting to note
that these values are significantly higher than expected for a typical precam-
brian shield.

Recently Kaminuma and Nagao [1981] reported the results of heat flow meas-
urements in Lutzow Holm Bay, near the margin of the East Antarctic Shield.
They measured the geothermal flux at five sites which had a maximum separation
of 40 km and found values ranging from 35 to 220 mw m\(^{-2}\). Heat flow values show-
ed high variability over even shorter distances, with the measured flux at two
Figure 13.

Derived values for the basal heat flux as a function of the exponent p for class A flow (dashed lines) and class B flow (solid lines). I, II, and III refer to the fixed average-value, step-change, and fixed current-value models respectively.
sites separated by less than 10 km being 35 and 110 mw m$^{-2}$. At four of the
five sites the measured flux is unexpectedly high (over 100 mw m$^{-2}$), but per-
haps results from the shield being thinner at its margin. However, because of
the high spatial variability it is difficult to characterize the geothermal
regime of the shield margin on the basis of only a few measurements.

Derived Ice Dynamics and Accumulation Rate Change-

Because our results show that the surface temperature warming is a smooth
monotonic function of the strain rate exponent $p$, it is apparent that if the
actual warming were known we could then infer the strain rate distribution
with depth. (There could be some ambiguity, as both class A and class B dyna-
mics generate surface warmings less than about 8 °C.) We need to know then how
the isotopic record in the ice sheet is related to the mean surface tempera-
ture at the time of deposition, and how this relation has changed as a func-
tion of time.

Over much of the Antarctic continent, the average surface isotope value
(usually averaged over ≈10 years) and the mean surface temperature (often the
10 m firm temperature) seem to be fairly well correlated. An analysis by
Lorius and Merlivat [1977] of data collected at sites along a traverse from
Dumont d'Urville toward Dome C found the average $\delta^{18}O$ values to be linearly
related to the mean surface temperature with the gradient $d\delta^{18}O/dT_s = 0.75$ °C$^{-1}$. A similar study by Morgan [1981] on data taken from sites over the en-
tire continent gives a comparable result: $d\delta^{18}O/dT_s = 0.84$ °C$^{-1}$.

Other studies have involved correlating the isotopic ratio in falling or
newly fallen snow with simultaneous surface temperatures [Gordiyenko and
Barkov, 1973; Kato, 1978], and temperatures at various levels in the atmos-
phere [Aldaz and Deutsch, 1967]. In general these time-series studies show the
isotope-surface temperature gradients to be smaller than those found in the
areal studies discussed above. It is interesting to note though that each type
of study seems to give consistent results: areal studies give a surface gradi-
ent of ≈ 0.8 °C$^{-1}$, while on the polar plateau, time-series studies give
$d\delta^{18}O/dT_s = 0.4$ °C$^{-1}$ when monthly average temperature and isotope values are
regressed.

It is unclear why these results should be so different. Perhaps post-de-
positional processes play an important role, but these effects would tend to
be smoothed out in an areal average which includes coastal and interior sites. Possibly in the single-year time-series studies we are seeing the effects of atmospheric processes not related to temperature, which are filtered out when multi-year averages are taken.

These considerations suggest that isotope-temperature gradients given by the areal correlations may be more appropriate here. In order to establish at least a tentative climatic scenario for the Dome C site, we will use the Lorius and Merlivat value, 0.75 % °C⁻¹ for the isotope-temperature gradient, although the slightly higher value suggested by Morgan won't effect our conclusions significantly. We are in effect assuming that the spatial correlation at a fixed time between surface temperature and the isotopic composition of surface firm is also true in the time domain for a fixed site. The current spatial correlation is certainly affected by factors such as atmospheric circulation patterns and source removal effects [Bromwich and Weaver, 1983]. While these processes are likely to have been different during the past glacial, their effects on the isotopic composition of snow on the polar plateau are not known. Thus we will also assume that isotope-temperature gradient we use here is time-independent.

To calculate the average isotopic shift from the late glacial to the early Holocene, we take the difference in the average prewarming δ¹⁸O value (i.e., average value from 870 to 510 m depth) and the average post-warming value (310 to 0 m depth). Corrected for the 1.6 % oceanic dilution effect, the net isotopic shift is 6.62 %, which differs slightly from the 7.0 % shift reported by Lorius et al. [1979], who calculate the shift starting from the late Wisconsinan Glacial Maximum phase at 670 m depth. By taking δ¹⁸O/δ¹⁸O_s of 0.75 % °C⁻¹, we find the mean surface warming implied by the 6.62 % oxygen isotope shift to be about 8.8 °C.

From figures 11 and 12, there are two class B strain-rate functions that give rise to an 8.8 °C warming and no class A functions. The class B functions have p=2.0 and p=2.5 corresponding to a=0.040 m a⁻¹ and a=0.044 m a⁻¹ respectively. The NSS model also has p=2.5, with a undergoing only a 5% increase from the beginning to the end of the climatic transition; from 0.0426 m a⁻¹ to 0.0445 m a⁻¹. Note that all these derived accumulation rates are nearly equal to the current value within measurement error. Thus for the simple strain rate functions we consider here and with our assumptions on the surface and basal boundary conditions, we find the accumulation rate at Dome C to be essentially
unchanged since the end of the Wisconsinan glacial. This also seems to be consistent with our assumption that the ice sheet thickness has remained essentially unchanged, although the effect of sea level rise on the ice sheet shape should also be taken into account (see Alley and Whillans, [1984]).

**Longitudinal Stress Distribution**

It is also interesting to calculate the longitudinal deviator stress distribution for the above strain rate distributions. We take the strain rate and deviator stress components to be related by Glen’s flow law

\[
\dot{\varepsilon}_{ij} = A(\theta) \tau^{n-1} \sigma_{ij}
\]  

(45)

Assuming the flow to be two-dimensional and ignoring shear, this reduces to:

\[
\dot{\varepsilon} = A(\theta) \sigma^n
\]  

(46)

where \(\sigma\) is the longitudinal deviator stress and we take \(n=3\). For \(A(\theta)\), we use the results of a study by Hooke [1981] who analyzed the work of a number of experimenters and found that empirically:

\[
A(\theta) = A_0 \exp\left[{-Q/RT + 3C/(T_r-T)^k}\right]
\]  

(47)

where \(A_0 = 1.39 \times 10^{14}\) a bar\(^{-3}\), \(Q=78.8\) kJ mol\(^{-1}\), \(C=0.16612\) \(\circ\)K\(^{-1}\), \(k=1.17\), \(T_r=273.39\) K, and T is the temperature in Kelvins. For thin-skin dynamics, we have from equations (46) and (47):

\[
\sigma = [(p+1)\dot{a}(1-\xi)^p/Ha(\theta)]^{1/3}
\]  

(48)

In figure 14 we have plotted \(\sigma\) as a function of depth for the best-fit model \(p=2.5\), \(\dot{a}=0.044\) m a\(^{-1}\), and also for \(p=0\) (constant strain rate) and \(p=1\). Note the important role temperature plays in the flow as in each case \(\sigma\) decreases more rapidly with depth than expected. For example, for \(p=0\), the deviator stress is not independent of depth as would be the case if \(A(\theta)\) were constant. It is interesting that the calculated stresses are very similar in the lower two-thirds of the ice sheet for all three cases, so that the stress in this zone seems to be only weakly dependent on the strain rate profile.
Figure 14.

Longitudinal deviator stress as a function of depth for the thin-skin strain rate function $p=2.5$, $\dot{a}=0.044 \text{ m a}^{-1}$ (solid line). Also shown are the derived stress profiles for $p=1$, $\dot{a}=0.039 \text{ m a}^{-1}$, and $p=0$, $\dot{a}=0.036 \text{ m a}^{-1}$. 
Discussion—

In figure 15 we compare the annual layer thickness as a function of depth predicted by these best fit models with the layer thicknesses measured by Thompson et al. The data show layers thinning slowly with depth, unlike the rapid thinning characteristic of class B flow. Only for $p=0$ are layers thinned slowly enough with depth to be consistent with the data (see figure 9). As discussed earlier, in our model a constant strain rate results in no warming or a small cooling, implying that the pronounced glacial-interglacial shift in the oxygen isotope ratio may not be simply related to the shift in mean surface temperature.

The Thompson et al. study measured annual layer thicknesses by identifying peaks in the microparticle concentration in various sections of the 900 m Dome C core. These microparticle peaks were assumed to be annual, as near the surface where layer thinning due to ice flow is unimportant, derived accumulation rates were close to the current measured value. However, pit studies by Palais et al. [1982] indicate that the near surface microparticle stratigraphy does not in general show well-defined peaks, but that annual features can be identified if other stratigraphic information is available, such as the visible stratigraphy. A few well-defined peaks with amplitudes several times larger than the background do occur in the upper 3 meters of firn, with a temporal spacing on the order of 5 to 10 years. Thus it may be possible that the microparticle peaks seen in some of the core sections are not annual, but represent on the average some longer period. This would make observed annual layers thinner, in better agreement with our predictions here.

If thin-skin dynamics are operative, then the length of the available climatic record could be longer than previously thought. In figure 16 we plot the depth-age relations for the ice dynamics inferred here along with the chronology calculated by Lorius et al. using a constant strain rate. All the models give similar ages in the upper 600 meters. However, by 1400 meters depth, the thin-skin strain-rate functions derived here imply ages nearly 20,000 to 30,000 years older. This ice may have been deposited during the Sangamon interglacial. It is difficult to speculate on the age of the ice much deeper, since several of our assumptions may no longer be true, such as the invariance of the ice sheet thickness, the location of the ice divide in relation to Dome C, and the absence of significant basal melting.

45
Figure 15.

Annual layer thickness as a function of depth for best-fit temperature profiles, for a surface warming of 8.8 °C. For curve A: \( p=2.0B, \Delta = 0.040 \text{ m a}^{-1} \); curve B: \( p=2.5B, \Delta = 0.044 \text{ m a}^{-1} \); curve C: \( p=2.5B, \) step-change model (see table 3 for values). Shaded region shows range of maximum and minimum layer thicknesses as a function of depth.
Figure 16.

Depth-age relations at Dome C. Closed dots give depth-age relation based on measured average layer thickness, and dashed line gives depth-age curve from Lorius, et al. [1979]. Curve A and B are calculated for class B models $p=2.0$, $\dot{a}=0.040 \, \text{m} \, \text{a}^{-1}$, and $p=2.5$, $\dot{a}=0.044 \, \text{m} \, \text{a}^{-1}$, respectively.
The question remains as to whether the strain rate distribution we derive here is unique. That is, do other functions exist that satisfy the our boundary conditions (namely that the strain rate be a well-behaved monotonic function of depth and vanish at the base) and for our interpretation of the Dome C climatic transition data, also give acceptable temperature profiles? Certainly there are a large number of functions which satisfy the above boundary conditions and to test all the potential candidates using the approach described in this study would be very time-consuming. We expect, however, any other function that generates a good temperature fit to be very similar to those derived here.

Another important question is how our results change if different boundary conditions for the ice dynamics are chosen. Does basal melting occur and is the ice sheet sliding as a result? Does the strain rate profile vary monotonically with depth or is there some marked discontinuous change in the ice rheology because of, say, sharp changes in impurity content or crystal orientation? Of course, these boundary conditions could be incorporated in our program, but this would involve the specification of additional parameters (such as basal melt rate, sliding velocity, location of strain rate extrema or discontinuities) that are virtually unknown.

Based on these considerations, it may be unrealistic to claim that we have derived the actual stress, strain rate, and velocity distributions at Dome C. However, we have shown the critical role ice dynamics plays in the reconstruction of the climatic history at a site. Because of the influences both climate and ice flow have on the temperature distribution, measuring this parameter at a borehole site provides a powerful constraint on the climatic interpretation of the ice core stratigraphy and proposed ice flow models.

SUMMARY—

We first list our major assumptions:

(1) The ice thickness is constant at its current value, 3400 m.

(2) The temperature distribution in the ice sheet prior to the climatic transition is a steady-state distribution.

(3) There is no sliding at the ice sheet base, and even though the basal temperature is taken to be at the pressure melting point, bottom melting can be ignored to the extent that the vertical velocity vanishes at the base.
(4) The vertical strain rate is a simple monotonic function of depth and the depth variation is proportional to \((1-(z/H))^P\) (laminar-like flow) or \((1-z/H)^P\) (thin-skin flow).

(5) Three general climatic transition models are considered, each characterized by the assumed time dependence of the accumulation rate. In all three cases, the start and end of the Wisconsinan-Holocene transition is assumed to correspond to the age of the ice at 510 and 380 m depth respectively, as suggested by the Dome C oxygen isotope profile. For the fixed-current-value model, the accumulation rate is held constant at the current value, \(0.040 \text{ m a}^{-1}\). In the fixed-average-value model, the accumulation rate is constant, but is adjusted for each strain rate function to give an age of 15,500 years at 510 m depth. Finally, for the step-change model, the accumulation rate is adjusted for each strain rate function to give ages of 15,500, 12,600 and 10,500 y.b.p. at depths of 510, 437, and 380 m respectively, as suggested by the correlation of the oxygen isotope profiles from the Dome C core and a marine sediment core from the Indian Ocean. An average value is calculated for each time interval 0-10,500 y.b.p., 10,500-12600 y.b.p., and 12,600-15,500 y.b.p., with the latter value also used to calculate the prewarming steady-state temperature distribution.

(6) We also assume that the oxygen isotope ratio in polar precipitation is linearly related to mean surface temperature, and this relation has remained unchanged through time. For the gradient \(d\delta^{18}O/d\delta^g\), we use the value 0.75 \(\%\ \text{C}^{-1}\), taken from the Dumont d'Urville-Dome C traverse data.

The shift in the stable isotope ratio over the climatic transition then corresponds to an 8.8 °C warming. Our calculations show that the strain-rate functions that generate this warming are thin-skin rather than laminar-like flow, with \(p=2\) to 2.5, depending on the climatic transition model. The fixed average accumulation rate and the step-change accumulation rate are close to the current measured value and imply at most only about a 5% increase since the end of the Wisconsinan glacial.

One consequence of thin-skin dynamics is that annual layers are thinned rapidly with depth, and as a result, the available climatic record may extend back further than previously thought.
APPENDIX

In this appendix we calculate the effect of densification on the vertical velocity in the upper part of the ice sheet. Our approach is basically the same as that of Paterson [1976].

For firn densifying in an ice sheet undergoing strain, we assume the firn densifies passively on top of the ice. Then the horizontal movement of the firn is the same as the ice beneath it. Assume the ice sheet is close to steady-state. Then from continuity:

(A-1) \( \nabla \cdot (\rho \mathbf{v}) = 0 \)

where our coordinate system is the same as that in the text. If horizontal variations in density can be ignored, the above equation becomes:

(A-2) \( \rho \frac{\partial v_x}{\partial x} + \rho \frac{\partial v_y}{\partial y} + \frac{\partial (\rho v_z)}{\partial z} = 0 \)

Using the definition of the strain rate components and the fact that ice is incompressible leads to:

(A-3) \( \frac{\partial v_z}{\partial z} + (v_z/\rho) \frac{\partial \rho}{\partial z} - \dot{\varepsilon}_z = 0 \)

The first term in the above equation can be thought of as the total strain rate, that is the strain rate resulting from the combined effects of densification and deformation. In equation (A-3), \( \dot{\varepsilon}_z \) is the strain rate in ice, taken at the true depth below the surface. This equation may be solved by use of an integrating factor with the result:

(A-4) \( v_z(z) = -(1/\rho) \int \dot{\varepsilon}_z(z') \rho(z') dz' + c \)

Requiring that the vertical velocity at the surface be equal to the true thickness of the annual layer we have finally:

(A-5) \( v_z(z) = (\rho_1/\rho(z)) [\dot{a} - (1/\rho_1) \int_0^z \dot{\varepsilon}_z(z') \rho(z') dz'] \)

where \( \dot{a} \) is the annual accumulation rate in meters of ice and \( \rho_1 \) is the density of ice.
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